Starred questions are suitable for students aiming for a credit or higher.

1. Whether or not you agree with the hypotheses or conclusions, are the following syllogisms valid?
   (a) It’s bad to be depressed. It’s depressing having unstructured Algebra and Logic lectures. Therefore it’s good to have structured Algebra and Logic lectures.
   (b) People either live in Melbourne or Sydney. It’s bad to be depressed. It’s depressing living in Melbourne. Therefore it’s good to live in Sydney.

2. Decide whether the following well-formed formulae are (semantic) theorems:
   (a) \((P \Rightarrow \sim P) \Rightarrow P\)
   (b) \((P \Rightarrow \sim P) \Rightarrow \sim P\)
   (c) \(\left( (P \Rightarrow Q) \land (R \Rightarrow P) \right) \Rightarrow (\sim Q \Rightarrow \sim R)\)
   (d) \(\left( (P \Rightarrow Q) \land (R \Rightarrow P) \right) \Rightarrow (\sim R \Rightarrow \sim Q)\)
   *(e) \(\left( (P \lor Q) \land (Q \Rightarrow \sim P) \right) \Rightarrow Q\)
   *(f) \(\left( (P \lor Q) \land (Q \Rightarrow \sim P) \right) \Rightarrow P\)
   *(g) \(\left( (P \lor Q) \land (\sim Q \Rightarrow \sim P) \right) \Rightarrow Q\)

3. Recall that \(\mathbb{N} = \{0,1,2,\ldots\}\) and \(\mathbb{Z}^+ = \{1,2,\ldots\}\). Find explicit bijections
   (a) \(f : \mathbb{N} \to \mathbb{Z}^+\).
   (b) \(g : \mathbb{Z}^+ \to \mathbb{N}\).
   (c) \(h : \mathbb{Z} \to \mathbb{N}\).
   (d) \(k : \mathbb{Z}^+ \to \mathbb{Z}\).

4. In lectures we constructed a bijection \(f : \mathbb{Z}^+ \to \mathbb{Q}^+\). Use this to construct a bijection \(g : \mathbb{Z}^+ \to \mathbb{Q}\).

*5. If \(S\) is a set with a binary operation \(*\), then we say \(*\) is associative if
   \[\forall a,b,c \in S \ (a * b) * c = a * (b * c) \, .\]
   Carefully prove that addition and multiplication modulo \(n\) are associative binary operations on \(\mathbb{Z}_n\).

*6. Interpret the following statements in simple words, and determine whether they are true when the universe of discourse is \(\mathbb{Z}, \mathbb{R}, \mathbb{R}^+, \mathbb{C}, \mathbb{Z}_7\) or \(\mathbb{Z}_8\).
   (a) \((\forall x)(\exists y) \ x^2 = y\) \quad (b) \((\forall x)(\exists y) \ x = y^2\)
   (c) \(\sim\left[(\forall x)(\exists y) \ x = 2y\right]\) \quad (d) \((\forall x)[x \neq 0 \Rightarrow (\exists y) \ xy = 1]\)
   (e) \((\exists x)(\exists y) \ x \neq 0 \land y \neq 0 \land xy = 0\)