Starred questions are suitable for students aiming for a credit or higher.

1. Revisit the following argument from an earlier exercise sheet, but use the formal rules of the Predicate Calculus to prove that it is valid.

All of the students in my class are fierce or friendly. If a student is friendly then he or she is smiling. I can see students in my class who are not smiling. I conclude that my class contains fierce students.

*2. Find the flaws in the following arguments, and also give semantic reasons why the conclusion cannot follow from the hypothesis in each case:

(a)

1. (1) $(\exists x)(\exists y)F(x, y)$  
2. (2) $(\exists y)F(a, y)$  
3. (3) $F(a, a)$  
3. (4) $(\exists x)F(x, x)$  
2. (5) $(\exists x)F(x, x)$  
1. (6) $(\exists x)F(x, x)$

(b)

1. (1) $(\forall y)(\exists x)F(x, y)$  
1. (2) $(\exists x)F(x, a)$  
3. (3) $F(b, a)$  
3. (4) $(\forall y)F(b, y)$  
3. (5) $(\exists x)(\forall y)F(x, y)$  
1. (6) $(\exists x)(\forall y)F(x, y)$

3. Use the rules of deduction of the Predicate Calculus to prove the following sequents:

(a) $(\exists x)F(x, x) \vdash (\exists x)(\exists y)F(x, y)$
(b) $(\forall x)(\forall y)(\forall z)F(x, y, z) \vdash (\forall z)(\forall y)(\forall x)F(x, z, y)$

* (c) $(\forall x)(\exists y)(\forall z)F(x, y, z) \vdash (\forall x)(\forall y)(\exists z)F(x, y, z)$

*4. Let $R$ be a relation symbol and consider the wffs

$W_1 = (\forall x) \sim R(x, x) , \quad W_2 = (\forall x)(\forall y)(\forall z) \left( [R(x, y) \land R(y, z)] \Longrightarrow R(x, z) \right) \quad W_3 = (\forall x)(\forall y)(\exists z) [R(x, z) \land R(y, z)].$

(a) Find a model for $W_1, W_2, W_3.$
(b) Prove that no finite model exists for $W_1, W_2, W_3.$