Information for MATH3066 Algebra and Logic

Websites

It is important that you check both the MATH3066 website and the Senior Mathematics website regularly:


   MATH3066 webpage: http://www.maths.usyd.edu.au/u/UG/SM/MATH3066

Important announcements relating to Senior Mathematics are posted on the Senior Mathematics page. On the MATH3066 page you will find online resources and other useful links, updated regularly by the lecturer.

Lectures

<table>
<thead>
<tr>
<th>Times</th>
<th>Location</th>
<th>Lecturer</th>
<th>Consultation</th>
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<tbody>
<tr>
<td>10 am Mon, Wed, Thu</td>
<td>to be advised</td>
<td>David Easdown, Carslaw 619</td>
<td>1–2 pm Wed</td>
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Lectures run for 13 weeks, and the last lecture will be on Thursday 2 June.

Tutorials and Exercise Sheets

Tutorials (one per week) start in Week 2, currently scheduled for 1 pm on Mondays and 9 am on Wednesdays and Thursdays. You should attend the tutorial given on your personal timetable. Exercise sheets for a given week should be available from the MATH3066 webpage. Solutions should be posted on the webpage towards the end of any given week.

Assessment

Your final raw mark for this unit will be calculated as follows:

   15%: First Assignment due Week 7.
   5%: Peer Review of First Assignment in Week 8.
   15%: Second Assignment due Week 12.
   5%: Peer Review of Second Assignment in Week 13.
   60%: Exam at end of semester 1.

There is one examination of two hours duration during the examination period at the end of the semester. Further information about the exam will be made available from the webpage at a later date. Final grades will be returned within one of the following bands: 

High Distinction (HD), 85–100: representing complete or close to complete mastery of the material; Distinction (D), 75–84: representing excellence, but substantially less than complete mastery; Credit (CR), 65–74: representing a creditable performance that goes beyond routine knowledge and understanding, but less than excellence; Pass (P), 50–64: representing at least routine knowledge and understanding over a spectrum of topics and important ideas and concepts in the course.
References and notes
Notes of various kinds, some printed, some handwritten, will be posted regularly on
the MATH3066 website. The following are excellent books available in the Library and
relevant to topics in this course:

John B. Fraleigh. *A First Course in Abstract Algebra*. Scitech 512.02 2A.
P. Halmos. *Naive Set Theory*. Scitech 510.1 87 and 512.817 116

Fraleigh is a classic introduction to algebra, and Lemmon provides an excellent intro-
duction to the Propositional and Predicate Calculi. Halmos is a classic introduction to
modern set theory, quite remarkable in the way it synthesises the main ideas, and goes
well beyond the scope of this course.

Aims and Learning Outcomes
The aim of this unit of study is to introduce, illustrate and formalise some of the most
important underlying ideas behind modern algebra and mathematical logic. The topics
should draw from, and reflect upon, students’ mathematical experiences across all of the
main fields of mathematics, including number theory, algebra and analysis. By the end
of the semester, students should

- be fluent in analysing and constructing logical arguments;
- be conversant with the Propositional and Predicate Calculi, and related notions of
  syntax (deduction) and semantics (completeness);
- be informed about the historical underpinnings of abstract algebra that lead to
  axiomatic theories of groups, rings, integral domains and fields, and their use in
  exploring questions of decidability or impossibility;
- be fluent with a range of standard and exotic arithmetics and ring constructions,
  and be able to prove elementary propositions and theorems about them;
- have some appreciation of the Halting Problem and Turing’s use of it to prove the
  undecidability of first order logic.

Week-by-week outline

<table>
<thead>
<tr>
<th>Week</th>
<th>Topics</th>
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<tbody>
<tr>
<td>1</td>
<td>Introduction to logic and impossibility. Mathematical Implication. Truth tables.</td>
</tr>
<tr>
<td>3</td>
<td>Polynomials and factorisation. Introduction to groups, rings and fields.</td>
</tr>
<tr>
<td>4</td>
<td>Deduction and proof in the Propositional Calculus.</td>
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<tr>
<td>5</td>
<td>Metaprotocols. Soundness in the Propositional Calculus</td>
</tr>
<tr>
<td>6</td>
<td>Syntax and semantics. Completeness of the Propositional Calculus.</td>
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<tr>
<td>7</td>
<td>Quantifiers. Predicate Calculus. <strong>First assignment due.</strong></td>
</tr>
<tr>
<td>8</td>
<td>Deduction and proof in the Predicate Calculus. <strong>Peer review.</strong></td>
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<tr>
<td>9</td>
<td>Soundness and completeness of the Predicate Calculus.</td>
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<tr>
<td>10</td>
<td>Quotient constructions. Field extensions. Irreducible polynomials and finite fields.</td>
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<tr>
<td>11</td>
<td>Construction of the real numbers and the completeness axiom.</td>
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<tr>
<td>12</td>
<td>Unsolvability of ancient problems of the Greeks. <strong>Second assignment due.</strong></td>
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<tr>
<td>13</td>
<td>The Halting Problem and undecidability of First Order Logic. <strong>Peer review.</strong></td>
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