Semester 1, 2020 (Main Exam)

The University of Sydney

MATH3066: Algebra and Logic

Lecturer: Oded Yacobi

Instructions (please read carefully and thoroughly):

1. **Exam Duration:** This exam will be released at 11am on Thursday 4 June 2020, and you have until 2pm on Thursday 4 June 2020 to complete, scan (or photograph), and upload your solutions to the CANVAS site.

2. **Preparing and submitting your script:** Please write your solutions legibly with either pen on paper, or on a tablet, or you may typeset them if you wish. Then combine your solutions into to a single (pdf or jpg) file. Then upload this pdf file to the dedicated canvas examination site for this unit.

   The 2pm deadline is STRICT. You must have your solutions submitted as a single (PDF or JPG) file to CANVAS by 2pm. Late submissions are NOT ACCEPTED.

3. **Exam format:** This exam consists of 6 and is worth a total of 120 points.

4. **Permitted materials:** This is an open book exam. The complete list of permitted material is:
   (a) lecture notes;
   (b) tutorials (including solutions);
   (c) assignments (including solutions);
   (d) notes that you have prepared throughout the semester in your studies.
   (e) past exams

   Accessing other material is forbidden, and is a form of plagiarism.

   Your are not permitted to use internet search engines or textbooks, or talk/chat/email with any other person for the duration of the exam.

5. **General:** You should show all relevant working. You may quote, without proof, any result from lectures unless you are explicitly asked to prove that result. Please state when you are applying theorems from lectures.

6. **Distribution:** The content of this exam is copyright to the University of Sydney, and must not to be shared or distributed in any form.

The Rules of Deduction:

(1) **Rule of Assumptions (A):** Any wff may be written down as an assumption, depending only on itself.

(2) **Modus Ponens (MP):** Given V and $V \Rightarrow W$, we may deduce W, depending on the pooled assumptions for V and $V \Rightarrow W$.

(3) **Modus Tollens (MT):** Given $\sim W$ and $V \Rightarrow W$, we may deduce $\sim V$, depending on the pooled assumptions for $\sim W$ and $V \Rightarrow W$.

(4) **Double Negation (DN):** Given $\sim\sim W$, we may deduce W, and vice-versa, in each case depending on the same underlying assumptions.

(5) **Conditional Proof (CP):** Given V, introduced earlier by Rule of Assumptions, and given W, relying on V, we may deduce $V \Rightarrow W$, discharging V, but relying on any remaining assumptions used to deduce W from V.

(6) $\wedge$**-Introduction ($\wedge$ I):** Given V and W, we may deduce $V \wedge W$, relying on the pooled assumptions for V and W.

(7) $\wedge$**-Elimination ($\wedge$ E):** Given $V \wedge W$, we may deduce V or deduce W, relying on assumptions for $V \wedge W$.

(8) $\vee$**-Introduction ($\vee$ I):** Given V, we may deduce $V \vee W$ or deduce $W \vee V$ for any W, relying on the assumptions for V.

(9) $\vee$**-Elimination ($\vee$ E):** Given $V \vee W$ and two deductions of C, firstly from V, introduced by Rule of Assumptions, and secondly from W, introduced by Rule of Assumptions, we may deduce C again, but from $V \vee W$, discharging the assumptions V and W, but pooling any assumptions for $V \vee W$ and any assumptions used to deduce C from V and C from W.

(10) **Reductio ad Absurdum (RAA):** Given V, introduced earlier by Rule of Assumptions, and given the contradiction $W \wedge \sim W$, relying on V as an underlying assumption, we may deduce $\sim V$, discharging the assumption V, but relying on remaining assumptions used to deduce $W \wedge \sim W$ from V.

(11) $\forall$**-Introduction ($\forall$ I):** Given a wff $W(b)$, where b is a constant symbol that occurs at least once, we may deduce $(\forall x)\, W(x)$, where x is a new variable that does not appear in $W(b)$ and replaces b uniformly throughout $W(b)$, relying on the assumptions for $W(b)$, provided the symbol b does not appear in any wff in this list of underlying assumptions.

(12) $\forall$**-Elimination ($\forall$ E):** Given a wff $(\forall x)\, W(x)$, we may deduce $W(b)$, where b is a constant symbol replacing x uniformly throughout $W(x)$, relying on assumptions for $(\forall x)\, W(x)$.

(13) $\exists$**-Introduction ($\exists$ I):** Given a wff $W(b)$, where b is a constant that occurs at least once, we may deduce $(\exists x)\, W(x)$, where $W(x)$ results from $W(b)$ by replacing at least one occurrence of b by x, relying on assumptions for $W(b)$.

(14) $\exists$**-Elimination ($\exists$ E):** Given a wff $(\exists x)\, W(x)$ and a deduction of C from $W(b)$, introduced by Rule of Assumptions, where b is a new constant symbol that replaces x uniformly throughout $W(x)$, we may deduce C again, but from $(\exists x)\, W(x)$, discharging the assumption $W(b)$, but pooling any assumptions for $(\exists x)\, W(x)$ and any assumptions used to deduce C from $W(b)$, provided b does not appear in C or in any of these underlying assumptions.

THIS IS THE LAST PAGE OF THE EXAMINATION PAPER