Tutorial 1

1. Three fair coins are tossed. What is the probability that at least one coin lands heads? How much information is conveyed by the failure of this event?

2. At a certain university 40% of Arts students are male. Out of the male Arts students 10% smoke, while 25% of female Arts students smoke. What is the probability that an Arts student who smokes is male? Hint: Write down a table showing the joint probability distribution of the outcomes male/female and smoker/non-smoker.

3. What is the derivative of the function $-\log_2 x$ with respect to $x$?

4. Two dice are rolled, and all 36 possible outcomes are equally likely. What is the probability that the sum of the numbers shown by the dice is even?

5. An urn contains $r$ red balls and $b$ black balls. Two balls are drawn from the urn (and not replaced). What is the probability of the events:
   
   (i) A red ball and a black ball being drawn in that order,
   
   (ii) The balls are different colours,
   
   (iii) Two red balls are drawn.

   If the balls are known to be different colours, what is the probability that the first one is red?

6. The table below shows the joint probability distribution for the outcomes of two experiments with sample spaces $X$ and $Y$. Calculate the (marginal) probability distributions on $X$ and $Y$. Are the experiments independent? Calculate also the conditional probabilities $p(x_1|y_3)$ and $p(y_3|x_3)$.

<table>
<thead>
<tr>
<th></th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$x_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_1$</td>
<td>$\frac{1}{9}$</td>
<td>$\frac{1}{27}$</td>
<td>$\frac{1}{27}$</td>
<td>$\frac{1}{9}$</td>
</tr>
<tr>
<td>$y_2$</td>
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<td>$0$</td>
<td>$\frac{1}{9}$</td>
<td>$\frac{4}{27}$</td>
</tr>
<tr>
<td>$y_3$</td>
<td>$0$</td>
<td>$0$</td>
<td>$\frac{1}{9}$</td>
<td>$\frac{4}{27}$</td>
</tr>
</tbody>
</table>

7. There are four balls in an urn, labelled 1, 2, 3, 4. A single ball is drawn from the urn, with all equally likely to be chosen. Let $A$ be the event $\{1, 2, 3\}$ and $B$ the event $\{1, 2, 4\}$. Calculate $p(AB)$, $p(B|A)$ and $p(A|B)$. Given an example of a pair of not-impossible events that are independent.