Tutorial sheet: Week 3

Background: Section 2.1 – Elementary Market Model.

Exercise 1 What is the price at time 0 of a contingent claim represented by the payoff \( h(S_1) = S_1 \)? Give at least two explanations.

Exercise 2 Give a proof of the put-call parity relationship.

Exercise 3 Compute the hedging strategies for the European call and the European put in Examples 2.1.1 and 2.1.2.

Exercise 4 Consider the elementary market model with the following parameters: \( r = \frac{1}{4}, S_0 = 1, u = 3, d = \frac{1}{3}, p = \frac{4}{5} \). Compute the price of the digital call option with strike price \( K \) and the payoff function given by

\[
h(S_1) = \begin{cases} 1, & \text{if } S_1 \geq K, \\ 0, & \text{otherwise}. \end{cases}
\]

Exercise 5 Prove that the condition \( d < 1 + r < u \) implies that there is no arbitrage in the elementary market model.

Exercise 6 (MATH3965) Under the assumptions of Section 2.1, show that there exists a random variable \( Z \) such that the price \( x \) of a claim \( h(S_1) \) can be computed using the formula

\[
x = E_P(Zh(S_1))
\]

where the expectation is taken under the original probability measure \( P \). A random variable \( Z \) is then called a pricing kernel (note that it does not depend on \( h \)).

Hint: You may use the fact that the probability measures \( P \) and \( \tilde{P} \) are equivalent.