Tutorial sheet: Week 5

Background: Section 2.2 – Single-Period Market Models.

Exercise 1 Consider the market model $\mathcal{M} = (B, S)$ introduced in Exercise 4 (Week 4). We have $k = 3$, $r = \frac{1}{9}$, $S_0 = 5$. Moreover, the stock price $S_1$ is given by the following table

<table>
<thead>
<tr>
<th>$S_1$</th>
<th>$\omega_1$</th>
<th>$\omega_2$</th>
<th>$\omega_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\frac{60}{9}$</td>
<td>$\frac{40}{9}$</td>
<td>$\frac{30}{9}$</td>
</tr>
</tbody>
</table>

Are there any values for $K$ such that the call option $(S_1 - K)^+$ represents an attainable contingent claim?

Exercise 2 Consider the stochastic volatility model $\mathcal{M} = (B, S)$ introduced in Example 2.2.3 and assume that $0 \leq r < h$.

(a) Characterise the class of all attainable contingent claims in $\mathcal{M}$ and check whether the model $\mathcal{M}$ is complete.

(b) Describe the class $\mathcal{M}$ of all risk-neutral probability measures for $\mathcal{M}$.

(c) Describe the set of all arbitrage prices for the call option $(S_1 - K)^+$ where the strike $K$ satisfies $S_0(1 + l) < K < S_0(1 + h)$.

(d) (MATH3975) Assume that $r = 0$. Check directly whether the call option with strike $K$ such that $S_0(1 + l) < K < S_0(1 + h)$ is attainable and find the range of values of its arbitrage price.