MATH3961 – METRIC SPACES (Advanced)

Here are the expected learning outcomes; some key words and phrases may guide your revision. The most important concepts are capitalized; the least important of those mentioned are given in parentheses.

METRIC, triangle inequality, open ball, closed ball, (diameter, bounded set)
NORM, normed vector space, (ℓ_p^n, ℓ_p^∞, Hölder, Minkowski)
uniform norm on the space $\mathcal{B}(X, \mathbb{R})$ of bounded functions $f : X \to \mathbb{R}$
sequence, convergence, limit, closure
COMPLETE, completion
ISOMETRY, isometric embedding
(Lipshitz function)
Contraction Mapping Principle (application to ODEs)

OPEN, CLOSED, TOPOLOGY, discrete
dense, (nowhere dense, Baire's Theorem)
CONTINUOUS, homeomorphism, (uniformly continuous)
CONNECTED, disconnection, path-connected, (interval)
COMPACT, Heine-Borel Theorem, (separable, second countable, Lindelöf)
Applications of the above concepts to continuous functions $f : [a, b] \to \mathbb{R}$ on closed intervals, (Intermediate Value, Extreme Value Theorems, continuity implies uniform continuity)

HAUSDORFF, completely regular, (other separation properties, Urysohn's Lemma, Tietze extension Theorem)

HILBERT SPACE, hermitean inner product, perpendicular, orthonormal
orthonormal basis, Bessel's (In)equality, linear functional,
KEY EXAMPLES: ℓ_2^∞, $L^2(X, \mu_c)$
(omitted at first attempt:)
CBS INEQUALITY

For the Topology part of the course, the most important notions are (in descending order of importance): continuous, open, topology, compact, metric, Hausdorff. (This is not the logical order; for instance, "continuous" is defined in terms of open sets.)