

## MATH3961 – METRIC SPACES (Advanced)

Here are some key words and phrases to guide your revision. The most important concepts are capitalized; the least important of those mentioned are given in parentheses.

METRIC, triangle inequality, open ball, closed ball, (diameter, bounded set)

NORM, normed vector space, ( $\ell_p^n$ ,  $\ell_p^\infty$ , Hölder, Minkowski)

uniform norm on the space  $\mathcal{B}(X, \mathbb{R})$  of bounded functions  $f : X \rightarrow \mathbb{R}$

sequence, convergence, limit, closure

COMPLETE, completion

ISOMETRY, isometric embedding

(Lipshitz function)

Contraction Mapping Principle (application to ODEs)

OPEN, CLOSED, TOPOLOGY, discrete

dense, (nowhere dense, Baire's Theorem)

CONTINUOUS, homeomorphism, (uniformly continuous)

CONNECTED, disconnection, path-connected, (interval)

COMPACT, Heine-Borel Theorem, (separable, second countable, Lindelöf)

Applications of the above concepts to continuous functions  $f : [a, b] \rightarrow \mathbb{R}$  on closed intervals, (Intermediate Value, Extreme Value Theorems, continuity implies uniform continuity)

HAUSDORFF, completely regular, (other separation properties, Urysohn's Lemma, Tietze extension Theorem)

HILBERT SPACE, hermitean inner product, perpendicular, orthonormal

orthonormal basis, Bessel's (In)equality, linear functional,

KEY EXAMPLES:  $\ell_2^\infty$ ,  $L^2(X, \mu_c)$

(omitted at first attempt:)

CBS INEQUALITY

For the Topology part of the course, perhaps the most important notions are (in descending order of importance): continuous, open, topology, compact, metric, Hausdorff. (This is not the logical order; for instance, "continuous" is defined in terms of open sets.)