# MATH3968 Review Lecture

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**Review Lecture**

- Do the tutorial problems.
- Use the exercise sheets/do Carmo 1 and 2 as additional problem sources
- Check out the past exam papers (2002, 2003, 2005). http://www.library.usyd.edu.au/
- All but the first question of the 2005 exam are relevant.
- However, we have covered more material, which is not in these exams, and we covered some topics in more depth..
- *You may bring to the exam an A4 sheet, written on both sides, with whatever you want on it.* Typed or handwritten is okay, but you must create your own study sheet, not copy someone else's.

**What Types of Questions to Expect?**

Questions may include the following:

- Explicit, computational questions (eg 2005: 2, 3; 2003 1,6,7; 2002: 1,4,6,7).
- Applications of major Theorems (eg 2005: 4; 2003: 5; 2002:8)
- Questions that require some insight: like a number of your tutorial questions.
- Statements of theorems.
- Proofs/examples from class
- Me not explicitly listing a past question above does not mean it is less relevant.

There will appear on the exam

- (At least one) question which was on your tutorial sheets (could be a required or a recommended problem).
- (At least one) application of the Gauss-Bonnet theorem.

**Rough Summary Of Course**

Disclaimer: this list is not exhaustive! The exam may contain things that are not on it. The list of possible questions are by no means exhaustive; they are a place to start, NOT a list of all the things that you need to know.

**Curves**

- arc-length, curvature, signed curvature (in plane), torsion, Frenet frame, Frenet equations.
- Fundamental Existence and Uniqueness Theorem (proof not examinable)
- rotation index, total curvature, Theorem of Turning Tangents (proof not examinable)

Non-Exhaustive Question Possibilities:

- You could be asked to compute some of the above in an explicit example.

- You could be asked to show that a curve has a particular property.
- you could be asked to work with special types of curves, such as lines of curvature, asymptotic curves, geodesics.
- The Frenet equations are useful.

**General Analysis**

- open/closed sets in $\mathbb{R}^n$, continuous/smooth functions, homeomorphisms, diffeomorphisms
- differential of a smooth map
- Inverse, Implicit Function Theorems (proofs not examinable)
- regular/critical points and values

Non-Exhaustive Question Possibilities:
These are mostly just definitions, although these two theorems are very useful; know their statements.

**Surfaces in $\mathbb{R}^3$**

- regular surface, surface given as a graph , locally all surfaces given this way, surfaces of revolution, surfaces given as the pre-image of a regular value
- smooth functions, tangent plane, differential of a map
- first fundamental form (metric)
- area, orientation
- Gauss map, second fundamental form, normal curvature, principal curvatures
- Relationship between $dN$, $I$ and $II$.
- Gauss and mean curvature

Non-Exhaustive Question Possibilities:
There is so much to compute here, and many possibilities for questions!

- Compute any of the above
- questions involving normal curvature, eg lines of curvature, asymptotic lines, umbilic points
- questions involving curves on surfaces
- minimal surfaces, fact that they are critical points (in fact local minimises) for area
- examples of minimal surfaces: catenoid ((only minimal surface of revolution) , helicoid

Non-Exhaustive Question Possibilities:
Mostly here you should know the theory and the examples.

- covariant derivatives, Christoffel symbols, Gauss and Codazzi-Mainardi equations (you do not need to memorise these equations, although it is possible that I could ask you to derive them)
- Expression for Christoffel symbols in terms of metric.
- Gauss's Theorema Egregium
- Fundamental existence and uniqueness theorem for surfaces (Bonnet)
- parallel transport, rotation of vectors under parallel transport
- Geodesics,
- algebraic value of covariant derivative, geodesic curvature

- uniqueness and existence theorem for Geodesics (proof not examinable)
- exponential map, geodesic polar coordinates, length-minimising property of Geodesics

Non-Exhaustive Question Possibilities:
Again, this is core material.

- compute Christoffel symbols
- use in some way the fact that the Gauss curvature is invariant under local isometries
- Give geodesic equations, find Geodesics
- compute parallel transport, formula for how angle changes
- geodesic curvature ties in with normal curvature; $k^2 = k_g^2 + k_n^2$; use this in some way.

The most important theorem we have covered in this course is:

- Gauss-Bonnet theorem (local and global), and its corollaries
- Poincare-Hopf
- Morse's theorem.

**Abstract Manifolds**

- abstract manifolds, and all the relevant definitions: eg smooth functions, tangent vectors (both definitions), differential, orientation
- Riemannian metric, covariant derivative, Christoffel symbols, Levi-Civita covariant derivative, Geodesics.
- tangent bundle
- Examples: $\mathbb{RP}^n$, $\mathbb{R}^2/\mathbb{Z}^2$, Klein bottle, hyperbolic space, tangent bundle.

Non-exhaustive possible questions:

- Check that you know definitions, and that you understand why they make sense.
- Examples: prove that something is a manifold, or is/is not orientable.
- Explicitly compute some of the above, such as covariant derivatives, geodesics.

Vote on exam consultation times.