

Tutorial Week 2

MATH3968: Differential Geometry

Semester 2, 2009

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"Lecture notes" refers to *Lecture Notes for Differential Geometry, MATH3968* by Nigel O'Brien.

"do Carmo" refers to *Differential Geometry of Curves and Surfaces*, by Manfredo do Carmo.

Required Problems

1. do Carmo §1.5 Q12 and Lecture Notes Exercise Set 2, Q5
2. Lecture Notes, Exercise Set 2, Q1bc, Q2, Q3
3. do Carmo §5.7 p404 Q1bc
4. Let A be an invertible $n \times n$ matrix with real entries.
 - (a) Show that the following are equivalent:
 - (i) $AA^t = I, A^tA = I$;
 - (ii) for all $v, w \in \mathbb{R}^n$, $\langle Av, Aw \rangle = \langle v, w \rangle$;
 - (iii) the columns of A form an orthonormal basis for \mathbb{R}^n ;
 - (iv) the rows of A form an orthonormal basis for \mathbb{R}^n .

Such matrices are called orthogonal, and we denote the set of all such matrices by $O(n)$.

- (b) Show that $O(n)$ is a subgroup of the group of invertible $n \times n$ matrices under matrix multiplication, i.e. it contains the identity, the product of any two orthogonal matrices is orthogonal, and the inverse of an orthogonal matrix is orthogonal.
- (c) Show that an orthogonal matrix has determinant either 1 or -1 .

An orthogonal matrix with determinant 1 is called special orthogonal; the set of such matrices is denoted $SO(n)$ and is also a group.

Recommended Problems

5. Lecture Notes, Exercise Set 1, Q6
6. do Carmo §1.5 p22 Q1cde
7. do Carmo §5.7 p405 Q5