MATH3971/4071

Unit outline

LECTURER

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Please include your name and student number in all communication and/or send from your official university email address. For questions of non-confidential nature please use the Ed Discussion Forum

Consultations: Thursday 12:00-13:00 in Carslaw 709 or by appointment

LECTURE NOTES

Lecture notes will be made available on CANVAS every week on Friday. You are also encouraged to make use of reference books but you do not need to buy any of these.

TUTORIALS

Tutorial questions for Week 1 are available on Canvas. Tutorial questions for the following weeks are included in the weekly lecture notes made available on the Friday of the week before the tutorial takes place.

There are two types of exercises: exercises scattered in the main text and exercises added at the end of the document. Exercises scattered in the text are easy and meant to help you to test your assumed knowledge and understanding of the new concepts. Exercises at the end of the document will be typically more complicated. Both types of exercises should be attempted and some of them will be discussed in class.

No hard copies will be distributed.

Tutorials are an integral part of the course. You can only learn mathematics by doing problems yourself, so attending tutorials is absolutely essential for performing well in the course.

ASSESSMENT

Two assignments (2x15%)

Group assignments are not permitted. You are encouraged to collaborate with others in solving the problems, but the work submitted must be your own!

Submission of assignment 1: 18/04/2019
Submission of assignment 1: 24/05/2019

The assignments must be submitted through the relevant submission box in Canvas, where they will be passed through the text matching software Turnitin (scanned copies of handwritten assignments are fine).

Late assignments are not accepted and no credit will be awarded. The bettermark principle applies to assignments.

Final Exam (70%)

There will be two-hour final exam. Only material covered in lectures and tutorials will be tested using questions addressing the outcomes.

For all assessments, the rules for special consideration/arrangement apply. The maximal possible extension is 7 days.

The final mark is determined by the following criteria

- High Distinction (HD), 85-100: representing complete or close to complete mastery of the material;
- Distinction (D), 75-84: representing excellence, but substantially less than complete mastery;
- Credit (CR), 65-74: representing a creditable performance that goes beyond routine knowledge and understanding, but less than excellence;
- Pass (P), 50-64: representing at least routine knowledge and understanding over a spectrum of topics and important ideas and concepts in the course.
- Fail (F), 0-49: representing rather limited understanding on a significant range of topics and concepts.

THE COURSE

Optimisation theory is an indispensable tool for an applied mathematician. The questions how to maximise your gain (or to minimise the cost) and how to determine the optimal strategy/policy are fundamental for an engineer, an economist, a doctor designing a cancer therapy, or a government planning some social policies. Many problems in mechanics, physics, neuroscience and biology can be formulated as optimisation problems.

Optimisation theory has many diverse applications and requires a wide range of tools but there are only a few ideas underpinning all this diversity of methods and applications. This course will focus on two of them. We will learn how the concept of convexity and the concept of dynamic programming provide a unified approach to a large number of seemingly unrelated problems.

Assumed knowledge

linear algebra: vector spaces, orthonormal bases, linear mappings and matrices, determinants, eigenvectors and eigenvalues;

analysis: functions of many variables, limits, continuity, partial derivatives, integrals;

ordinary differential equations: concept of solution to a system of differential equations, solving systems of linear differential equations;

probability theory: probability, conditional probability, random variables, distributions and densities, expected values and variances, Gaussian (normal) distribution,

Schedule of lectures

1. Introduction: motivating examples and mathematical formulation of control problems
2. Convex sets, convex cones and convex functions. Static optimisation problems. Legendre-Fenchel transform and duality.
3. Optimisation of functions with constraints and KKT conditions.
4. Optimal control of ODEs: controllability, observability, stabilisation.
5. Optimal control of ODEs: Pontriagin Maximum Principle
6. Optimal control of ODEs: Dynamic Programming and the Hamilton-Jacobi- Bellman equation
7. Martingales, optional stopping theorem
8. Controlled stochastic systems, Dynamic Programming Principle and martingale characterisation of the optimal payoff
9. Hamilton-Jacobi-Bellman equation and optimal feedback controls
10. Optimal stopping
11. Kalman filter and Linear-Quadratic Regulator with partial observation for stochastic differential equations
12. One-period games, saddle points and Nash equilibria.
13. Dynamic games

LEARNING OUTCOMES

By completing this unit you will be able to

- analyse static optimisation problems with constraints
- formulate deterministic and stochastic dynamic optimisation problems, that arise in scientific and engineering applications, as mathematical problems,
- understand the importance of convexity for optimisation problems, and use convexity to determine whether a solution to a given problem exists and is unique.
- check if a certain controlled system is controllable, observable, stabilisable,
- apply the Maximum Principle in order to solve real world control problems,
- formulate the Hamilton-Jacobi-Bellman equation for solution of dynamic optimisation problems and solve them in special cases,
- Explain the derivations of key theoretical results and discuss the role of mathematical assumptions in these derivations,

- use game theory to formulate optimisation problems with many competing payers
- identify important solvable classes of optimisation problems arising in finance, economics, engineering and insurance and provide solutions

BOOKS

Additionally to the lecture notes you might consult other textbooks and monographs on control theory.

Textbooks and lecture notes

Boyd S and Vandenberghe L: Convex Optimization. Cambridge University Press, 2004. This is an excellent book on static optimisation in finite-dimensional spaces.

All the following books are about *optimal control of systems evolving in time.*

Evans LC: An Introduction to Mathematical Optimal Control Theory Evans

Haurie A, Krawczyk JB and Zaccour G: Games and dynamic games. World Scientific 2012

Liberzon D: Calculus of variations and optimal control theory. A concise introduction. Princeton University Press, 2012. This is a very nice, simple but mathematically rigorous book focused on fundamental ideas of deterministic optimal control.

Ross K: Stochastic control in continuous time Ross

Books focused on applications of optimal control

Lenhart S and Workman JT: Optimal control applied to biological models. Chapman&Hall/CRC, 2007

Geering HP: Optimal Control with Engineering Applications. Springer 2007

Koon, W.S., Lo, M.W., Marsden, J.E., Ross, S.D. Dynamical Systems, the Three-Body Problem and Space Mission Design. Marsden Books, 2011

Schättler H. and Ledzewicz U.: Optimal control for mathematical models of cancer therapies. An application of geometric methods. Springer 2015

Weber TA: Optimal control theory with applications in economics. MIT Press, 2011

More mathematically advanced books

The next three books are excellent introductions to optimal control of determinstic and stochastic systems. They are mathematically more demanding.

Bäuerle N and Rieder U: Markov decision processes with applications to finance. Springer 2011

Bressan A and Piccoli B: Introduction to the mathematical theory of control. Springer 2015

Pham, Huyên Continuous-time stochastic control and optimization with financial applications. Springer 2009