The University of Sydney  
School of Mathematics & Statistics  
Applied Mathematics IV Honours 2007

Applied Mathematics Honours offers prospective students a unique learning experience. For the first time, students can devote themselves completely to a single area of study in a field in which they have already achieved success. The Honours year is the *icing on the mathematics cake*.

In 2007 the University will be offering 50 Honours Scholarships worth $5,000 each. For more details check out the web-site (and follow the links):

www.usyd.edu.au/honours/costs.shtml

Applied Maths offers several teaching units, described in more detail below, including core methods units and interesting and practical applied units. Many of these units are at the very edge of current research. The program is very flexible, allowing students to select units from other disciplines and even some third year advanced units they may have missed on the way through.

The ICE-EM/AMSI\(^1\) Summer School for 2007 will be held at Sydney University from 15th January to 9th February. Students may also select approved units from the Summer School as part of their course requirements. For details see: www.maths.usyd.edu.au/u/amsiss07/.

The course assessments include the best seven units of study from a selection of eight or more units and a thesis equivalent to three units of study. The thesis may be an essay, computational project or an original research treatise on any approved topic and is conducted under the supervision of a staff member. Some suggested topics are listed later in this document.

Students completing an Applied Mathematics Honours degree will obtain both technical expertise and modelling proficiency that is aggressively sought

\(^1\)International Centre of Excellence for Education in Mathematics/Australian Mathematical Sciences Institute
in many industries (e.g. the financial sector). The program also provides a track for those more academically inclined and who are considering a career in teaching and/or research.

**Proposed Course Units for 2007**

**ADVANCED OPTION PRICING**  
*Dr P.W. Buchen*  
AOP is a follow-on unit from the 2nd and 3rd year Financial Maths units. The course contains several chapters including:

- Gaussian random variables, stochastic calculus, the Black-Scholes option model, Black-Scholes extensions, binary options, dual-expiry and dual-asset options, barrier options, credit derivatives, lookback options, Asian options and multi-asset, multi-period exotics.

The chapter on stochastic calculus is central to modern derivative pricing and covers topics including: Brownian motion, martingales, stochastic integrals and stochastic DE’s, Itô’s Lemma, the Feyman-Kac formula and Girsanov’s Theorem. Specialised mathematical techniques are developed to price a host of exotic options in the ensuing chapters. These techniques include a combination of both stochastic and PDE methods for pricing derivatives in the Black-Scholes framework and will bring the student to the forefront of research. This unit is arguably the most advanced course of its type anywhere in the country.

**INTEGRAL TRANSFORMS AND ASYMPTOTIC METHODS**  
*Dr C.M. Cosgrove*  
*Integral Transforms.* This part of the course will cover the basic theory of Fourier, Fourier-cosine, Fourier-sine and Laplace transforms and their applications to the solution of boundary-value problems in partial differential equations. The course will begin with a proof of the Fourier Integral Theorem by way of the Riemann-Lebesgue Lemma and the Riemann Localisation Theorem, from which the various inversion formulae and associated auxiliary formulae can be deduced. Thereafter, it will be shown how to solve some important partial differential equations, such as the heat/diffusion equation, Laplace equation, Helmholtz equation, etc., either by directly feeding in the transformed boundary conditions, or via the method of Green’s functions. The Wiener-Hopf technique for solving half-range boundary value problems
will also be briefly surveyed. Time permitting, we will introduce the Mellin and Hankel transforms and their (very beautiful) application to the electrified disk problem. Throughout this course, students will need to be reasonably proficient in handling complex contour integrals and real multiple integrals.

Asymptotic Methods. Asymptotic analysis is one of the most frequently used tools in virtually every branch of pure and applied mathematics, being required whenever one wants quantitative information about a mathematical expression (e.g., definite integral, infinite series, solution of a D.E., iterated formula, $n!$, $\zeta(1/2 + it)$, etc.) containing a small or large parameter. The course begins with a brief survey of the great variety of problems and results in asymptotic analysis (only a few of which can be treated in 12 lectures) and some elementary methods based on manipulation of series and integration by parts. Next, the powerful Abel-Plana and Euler-Maclaurin summation formulae are introduced, with applications to the gamma/factorial function (the full Stirling’s formula with error term) and Riemann zeta function. About half the course will be devoted to the asymptotics of definite integrals containing a large parameter. Real integrals of exponential type are handled using Laplace’s method, and the underlying Watson’s Lemma will be proved. This theory will then be extended to the complex domain and the very powerful method of steepest descent paths and saddle points will be studied. Time permitting, other useful techniques such as stationary phase, Mellin transforms, WKB methods, etc., may get a brief mention.

Computational Projects in Applied Mathematics  Dr G. Gottwald
Lectures will be given on seven projects over the semester, from which the students will select three for detailed numerical study. The areas covered will be relatively diverse in nature; for example, some or all of the following will be included:

• Symplectic integration for Hamiltonian systems
• Data analysis using singular value decomposition
• Modelling the spread of disease
• Flux expulsion in MHD (magnetohydrodynamics)
• Solving PDE’s with FFT’s (fast fourier transforms)
• Models involving neural networks
Numerical solution of stochastic DE’s

Students will be assessed by their reports on their chosen three projects. Timetable slots for this unit of study will be a mixture of lectures and laboratory sessions. The first project will be compulsory, so that the practical work can get underway quickly. Following this there will be two groups of three projects, and students will choose to do one project from each of these two groups. Each group of projects will be introduced by a series of lectures, after which students will select a project and work on it, in the following laboratory sessions and in their own time. The programming language used will depend on the nature of the individual projects and the programming experience of students attending.

**Optimal Control and Differential Game Theory**  
*Dr D.J. Ivers*

No information on this unit is available at this time as DJI is currently on leave.

**Modern Asymptotics and Perturbation Theory**  
*Prof N. Joshi*

Differential equations model most natural phenomena we know. Yet their solutions can be notoriously difficult to understand. In place of “exact” solutions, a rich array of asymptotic methods have been developed to qualitatively understand the solutions. These are based on either the intrinsic variables or an external parameter in the problem being large or small. The field is vast and ranges from the fundamental theory of asymptotic expansions and perturbation methods, developed by Poincaré for the study of the solar system, to modern advanced techniques which deal with cases where conventional asymptotics fails. This course will include asymptotics of boundary layers, WKB and multiscale methods and modern techniques developed to model dendritic growth (such as snowflakes), fluid flow (existence of solitary waves), and the onset of chaos. The only background needed is the basic theory of differential equations and complex analysis.

**Geophysical Fluid Dynamics**  
*A/Prof C. Macaskill*

This course will give an introduction to motions in the atmosphere and oceans, which cover a huge range of space and time scales, from the detailed features of a turbulent surface wave breaking on the shore to the global scale of the ocean circulation. The course starts with the basic equations of fluid
dynamics in a rotating, density-stratified medium. Phenomena where rotation effects dominate will be covered first: vorticity dynamics and bottom effects, Rossby, Kelvin and inertia-gravity waves, simple models of ocean circulation. Secondly, waves that exist due to density variations and jumps will be discussed: linear and nonlinear surface waves, shallow water theory and the Korteweg-de Vries equation, internal gravity waves. The final part of the course will discuss the many cases where rotation and stratification both play significant roles, and explain the critical role of potential vorticity and the quasi-geostrophic equation. Applications will include instability of fronts and jets and the formation of vortices, interactions of vortices with coastlines and topography and turbulence in the oceans and atmosphere. Throughout the course simple MATLAB models will be used to supplement and illustrate many of the concepts and ideas.

**Populations and Disease**

*Dr M. Myerscough*

This course presents a more complex approach to the mathematics of populations and diseases than that of MATH3963. Topics that will be covered include age-structured populations, matrix models for populations and life-history graphs, the original Kermack-McKendrick model for infectious diseases, spatial spread of epidemics, travelling waves, epidemics in structured populations, stochastic models for diseases and anything else that roughly fits the theme, is interesting and that we can fit in.

The course will be very much more interactive than the third year course. Homework will be evenly distributed throughout the semester rather than in the form of one or two big assignments and will include a reading assignment in the current research literature which will be presented as a talk to the class.

**Mathematical Physiology**

*Dr M. Wechselberger*

Physiological rhythms are central for life. Prominent examples are the beating of the heart, the activity of neurons, or the release of hormones regulating growth and metabolism. All these rhythms have in common, that they evolve on at least two different time scales, i.e. there exist a quasi steady state of the system on a slow time scale (e.g. the resting state of the heart) interspersed by a dramatic change of the system on a fast time scale (e.g. the heartbeat itself). Mathematical models of such systems are called slow/fast systems or multiple scales problems.

In this unit we introduce a powerful mathematical technique suitable for
analysing such multiple scales problems called geometric singular perturbation theory. This perturbation theory is based on the fact that the system under study has a singular perturbation parameter and classical asymptotic analysis breaks down. The method is very powerful and is based on dynamical systems techniques like bifurcation theory and invariant manifold theory. We will develop the basic mathematical tools to analyse physiological problems.

The class of physiological problems we study is electrical signaling in excitable cells. We analyse the famous Hodgkin-Huxley model of the squid giant axon describing action potential propagation along a neuronal axon. We use reduction techniques introduced by FitzHugh to obtain a qualitative model of the neuron which can be fully analysed by geometric singular perturbation theory. This analysis will demonstrate how neurons can fire action potentials and therefore communicate information. Another problem of interest is electrical signaling in pancreatic cells which leads to secretion of insulin due to bursting electrical activity. The only background needed is the basic theory of differential equations and stability analysis (introduced in third year ODE courses).

**Suggested Projects for 2007**

1. **Dr Buchen**
   - Pricing CDO’s and other portfolio credit derivatives
   - The method of images in option pricing
   - Models of behavioural economics

2. **Dr Cosgrove**
   - The Kerr metric in general relativity
   - Gravitational collapse
   - Studies in integrable differential equations

3. **Dr Galloway**
   - Parameter study of ABC dynamos
   - Force-free magnetic fields in the Sun’s corona
   - Chaos in the solar system

4. **Assoc. Professor Gibson**
   - Neural networks and rat’s whiskers
Communication in glial cells
Functional magnetic resonance imaging

5. Dr Gottwald
   Critical pulse propagation in excitable media
   Networks of coupled oscillators
   Cardiac alternans
   Discrete stochastic processes and their asymptotic limits

6. Dr Ivers
   Core surface motions
   Thermal and magneto-convection
   Planetary and galactic dynamo theory
   $P$ and $HP$ finite element methods
   Protein folding
   Biological or quantum computing

7. Professor Joshi
   Nonlinear integrable difference equations
   Exponential asymptotics

8. Assoc. Professor Macaskill
   Vortex blob methods for vortex dynamics
   Tracking combustion waves using the CASL method from vortex dynamics
   Jet instability in the ocean: the role of boundaries and topography
   The interaction of internal gravity waves and vortical motions in the ocean

9. Dr Myerscough
   Modelling nest site selection in ants
   Simulating bee behaviour with individual-oriented models
   Immunological and epidemiological models

10. Dr Poladian
    Phylogenetic algorithms in molecular biology
    Artificial embryology in optimisation algorithms
    Level set theory for topological optimisation
    Algebraic varieties in phylogenetics
    Representation in genetic algorithms and other topics in this area
11. Dr Thompson  
Bubbles, cells and ultrasound  
Nonlinear acoustics  
Models for biomedical ultrasound  

12. Dr Wechselberger  
Relaxation oscillators, return maps, and devil’s staircases  

Rights and Privileges  
Applied Maths Honours students receive many privileges including:  

- office and desk space  
- computer account with email and TeX facilities  
- photo-copying account and laser printing access  
- after hours access key to the Carslaw building  
- class rep at school meetings  
- eligibility for many Honours awards and prizes  
- School social events  

Further Information  
For more information please consult the 2006 Applied Maths 4 Handbook at:  
www.maths.usyd.edu.au/u/UG/HM/#Handbooks  
The 2007 Handbook is currently under construction and will be available around the end of October. Much of the 2006 material will still be valid for 2007.  

The Applied Maths Honours coordinator for 2007 will be:  

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