1. The following table shows the average monthly production \( y \) of a certain commodity for the years 1948-1958.

<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>yield, ( y )</td>
<td>50.0</td>
<td>36.5</td>
<td>43.0</td>
<td>44.5</td>
<td>38.9</td>
<td>38.1</td>
<td>32.6</td>
<td>38.7</td>
<td>41.7</td>
<td>41.1</td>
<td>33.8</td>
</tr>
</tbody>
</table>

Construct a

(a) i. 5 year moving average.

ii. 4 year centred moving average for the data.

(b) Plot the ts and the moving averages in a(i) and a(ii).

2. Write down an expression for the sample autocorrelation coefficients at lags \( k \) \((r_k)\) based on \( N \) observations \( x_1, x_2, \ldots, x_N \). \((r_k \text{ is an estimate of } \rho_k \text{ (the true autocorrelation at lag } k)\).

3. Consider the time series given by:

\[
\begin{array}{cccccccccccc}
  t & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 \\
  x_t & 1.6 & 0.8 & 1.2 & 0.5 & 0.9 & 1.1 & 1.1 & 0.6 & 1.5 & 0.8 & 0.9 & 1.2 & 0.5 & 1.3 & 0.8 & 1.0 \\
\end{array}
\]

(a) i. Calculate the lag 1 sample autocorrelation coefficient, \( r_1 \) for the ts \( x_t \).

ii. Calculate sample autocorrelation coefficients at lags 2 \((r_2)\) and 3 \((r_3)\) respectively for the ts \( x_t \).

(b) Re-arrange the data as 15 pairs at lag 1 by filling the missing entries *:

\[
\begin{array}{cccccccccccc}
  & u_t = x_t & 1.6 & 0.8 & 1.2 & 0.5 & 0.9 & 1.1 & 1.1 & 0.6 & 1.5 & 0.8 & 0.9 & 1.2 & 0.5 & 1.3 & 0.8 \\
  & v_t = x_{t+1} & * & 1.2 & * & 0.9 & 1.1 & 1.1 & 0.6 & 1.5 & 0.8 & 0.9 & 1.2 & 0.5 & * & * & * \\
\end{array}
\]

i. Calculate the correlation coefficient, \( r \) between \( u_t \) and \( v_t \).

ii. Explain why \( r \) and \( r_1 \) are different.

4. Suppose that \( \{Z_t\} \sim WN(0, \sigma^2) \). Find the autocorrelation function, \( \rho_k \) of \( \{Z_t\} \). Plot this autocorrelation function \( \rho_k \) against \( k \).

5. Suppose that \( \{X_t\} \) is generated by \( X_t = Z_t - Z_{t-1} \), where \( \{Z_t\} \sim WN(0, \sigma^2) \) Find the autocorrelation function, \( \rho_k \) of \( \{X_t\} \) and plot this autocorrelation function \( \rho_k \) against \( k \).

6. Suppose that \( \{X_t\} \) is generated by \( X_t = Z_t + 0.8Z_{t-1} - 0.10Z_{t-2}(\ast) \), where \( \{Z_t\} \sim WN(0, \sigma^2) \)

(a) Find \( E(X_t)\) and \( Var(X_t)\). Show that \( E(X_t)\) and \( Var(X_t)\) are independent of time, \( t \).

(b) Find the autocorrelation function of \( \{X_t\} \) and deduce that \( (\ast) \) can be used to represent a time series.

7. Calculate \( E(X_t), var(X_t) \) and \( \gamma_k = Cov(X_t, X_{t-k}) \) for all \( k \) in terms of \( \sigma^2 = var(Z_t) \) for \( X_t - 5 = Z_t + 0.8Z_{t-1} + 0.6Z_{t-2} + 0.2Z_{t-4} \). Obtain the correlogram for this processes.

PTO FOR PRACTICAL 1
1. The following table shows the average monthly production of bituminous coal in millions of kilogrammes for the years 1948-1958.

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield</td>
<td>50.0</td>
<td>36.5</td>
<td>43.0</td>
<td>44.5</td>
<td>38.9</td>
<td>38.1</td>
<td>32.6</td>
<td>38.7</td>
<td>41.7</td>
<td>41.1</td>
<td>33.8</td>
</tr>
</tbody>
</table>

*Yield - Average monthly production (millions of kgs)*

Using R:

a) Enter and save the average monthly production data (row 2) in d.

b) Obtain the tsplot of this data using the command: `>ts.plot(d)`

c) Change the label on the x-axis as 1948, ... etc.

   **Commands:**
   ```
   >dl = ts(d, start =1948)
   > ts.plot (dl, xlab = “year”,
   ylab = “Average Monthly Production”).
   ```

d) Use the following command to give a title to your time series plot in (c)

   ```
   >title (“Time series of coal production”)
   ```

e) Comment on the time series.

2. Consider the data in Q1 (Ref Tutorial Q1). Construct following moving averages for this time series.

   a) 5 year moving averages and store in f1.

   b) 4 year centred moving averages and store in f2.

   Hint: Use the ‘filter’ command with suitable weights (see below).

3. Graph the moving averages (ma) in (a) and (b) in Q2 together with the original data using

   ```
   >ts.plot(d,f1,f2) or >ts.plot(cbind(d,f1,f2)).
   ```

4. On page three of course notes contains three sets of time series data (see the course website). Copy and paste the set 1 on your R worksheet following the commands:

   ```
   dat1 = scan()
   ```

   1: paste your data.

   Obtain the time series plot of this data and comment on this time series.

   Repeat the above steps for the remaining two time series data sets.

**Smoothing via Moving Averages**

*The filter command (in R) can be used to calculate moving averages.*

**Command:** `> filter (d, rep(1/ℓ, ℓ))` *(ℓ (odd) is the length(or span) of a simple ma)*

**Centered ma of span = ℓ (even)**

**Command:** `> filter (d, c(w1, w2, ..., wℓ+1))` ,

where $w_1, \ldots, w_{\ell+1}$ are the weights of a centered ma filter.

**Hint:** Restore S+help window and look at ‘filter’ for more details.