THE UNIVERSITY OF SYDNEY

Assignment 2 STAT3011/3911 Stochastic Processes 2008.

Due by 3 pm, Monday April 28, Carslaw 8th Floor, assignment box STAT3XXX.

1. $\{Z(t),\ t \geq 0\}$ is standard Brownian motion.

   (a) *Evaluate* $E(Z(s)Z(t)), s \leq t.$

   (b) Let $X(t) = Z(t) - tZ(1)$. *Evaluate* $\mathrm{Cov}(X(s), X(t))$, $0 \leq s, t \leq 1$.

2. Cars pass a point on a highway at a Poisson rate of one per minute. Five percent of the cars on the highway are Nissans (= "successes").

   (a) *What is* the probability that at least one Nissan passes by during an hour?

   (b) Given that ten Nissans have passed by in an hour, *what is* the expected total number of cars which has passed by in that time?

   (c) If 50 cars in total have passed by in an hour, *what is* the probability that five were Nissans?

3. A small barbershop operated by a single barber has room for at most two customers. Potential customers arrive in a Poisson stream of 3 per hour. Successive service times (that is, times to cut hair) are iid exponential, with mean service time 15 minutes.

   (a) *Find* the average number of customers in the shop.

   (b) *Find* the proportion of potential customers that enter the shop.

   (c) If the barber worked twice as fast, what proportion of customers would enter the shop?

4. Each individual in a population has an $expl(\lambda)$ time to splitting into two, or an $expl(\mu)$ time to dying, whichever comes first, these times being independently distributed. Individuals act independently of each other in this respect. Immigrants join the population in a Poisson stream, rate $\theta\,(> 0)$ over time $t \geq 0$. Let $X(t)$ be the population size at time $t \geq 0$.

   (a) *Show that* if there are i individuals in the population, time to a change in population size is $\sim expl(i(\lambda + \mu) + \theta)$.

   (b) *Hence obtain* the entries of the intensity matrix $Q_c = \{q_{ij}\}$ of the Markov process $\{X(t), t \geq 0\}$.

Please turn over

5. **STAT3911 only.**

   (a) $\{Z(t),\, t \geq 0\}$ is standard Brownian motion. *Compute* the mean and variance of $\int_0^1 t^{1/2} e^{-t}\, dZ(t)$.

   (b) i. Let U_i, $i = 1, 2, \cdots, n$ be iid with uniform distribution on $(0, t]$. *Find* $E(\max_i U_i)$.

   ii. Use the above to *write down* the conditional expectation $E(S_{N(t)}|N(t))$ where $\{N(t),\, t \geq 0\}$ is a Poisson process, and S_n is the time to the n-th arrival. **Hint:** Theorem 3.2.

6. **STAT3911 only.** A stock is presently selling at a price of \$ 50 per share. After one time period, its selling price will be either \$150 or \$25. An option to purchase stock at time 1, for \$100 per share, can be purchased now at cost \$ c per share.

   (a) *What* should c be in order for there to be no sure win?

   (b) If $c = 4$, *produce* a betting strategy to guarantee a sure win.

END