Additional note 11

Lecture 30

1. **Revision questions of L29**

   - What is sequential SS?

     Answer:
     Type I SS: SS_A, $\text{SS}_\text{B}|\text{A}$, $\text{SS}_\text{A:B}|\text{A,B}$ (sequential)
     Type II SS: $\text{SS}_\text{A}|\text{B}$, $\text{SS}_\text{B}|\text{A}$, $\text{SS}_\text{A:B}|\text{A,B}$
     Type III SS: $\text{SS}_\text{A}|\text{B,A:B}$, $\text{SS}_\text{B}|\text{A,A:B}$, $\text{SS}_\text{A:B}|\text{A,B}$ (orthogonal)
     where $\text{SS}_\text{A:B}|\text{A,B}$ say is $\text{RSS}_\text{A,B} - \text{RSS}_\text{A,B,A:B}$.

   - What are the two conditions for balance incomplete block design?

     Answer: 1. $n = bk = tr,\ k < t$ and 2. $\lambda(t-1) = r(k-1)$.

   - What is the 95% CI for $\alpha_A - \alpha_B$?

     Answer: $\frac{rk}{\lambda t}[(\bar{Y}_{A(\bullet)} - \bar{B}_A) - (\bar{Y}_{B(\bullet)} - \bar{B}_B)] \pm t_{n-p}(0.975)\sigma\sqrt{\frac{2k}{\lambda t}}$

2. Read the three-way ANOVA table to see how the different sum of squares can be calculated using different types of group totals. Note the way of splitting total SS as well as df into different order of interaction components within the treatment SS (TSS) and then RSS.

3. Note how different factorial models can be formulated in R.

4. When there are many factors to be tested for significance, 2^n factorial model can be applied with n factors each with 2 levels only to simply the model.

Lecture 31

1. **Revision questions of L30**

   - What is problem if we build a n order factorial model from a 2^n data without replicates?

     Answer: This is a saturated model and so there is no df for the residuals. In other words, the model will give perfect fit and the residuals are all 0.

   - What is the remedy for this problem?

     Answer: We should drop some higher order interaction terms so that these df will move to the residuals.

   - What is the standard order for a 2^4 factorial design?

     Answer: The standard order for the combination of high and low for factor A, B, C, D are
     1 a b ab c ac bc abc d ad bd abd cd acd bcd abcd
     A=factor(c(0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1)) gl(2,1,16,labels=0:1)
     B=factor(c(0,0,1,1,0,0,1,1,0,0,1,1,0,0,1,1)) gl(2,2,16,labels=0:1)
     C=factor(c(0,0,0,0,1,1,1,1,0,0,0,0,1,1,1,1)) gl(2,4,16,labels=0:1)
     D=factor(c(0,0,0,0,0,0,0,0,1,1,1,1,1,1,1,1)) gl(2,4,16,labels=0:1)

2. Note the difference between one-way ANOVA with fixed (type I) and random (type II) treatment effect.

   Fixed: $Y_{ij} = \mu + \alpha_i + \epsilon_{ij},\ j = 1,\ldots,n_i,\ i = 1,\ldots,t,\ \ \epsilon_{ij} \sim NID(0,\sigma^2)$

   Random: $Y_{ij} = \mu + A_i + \epsilon_{ij},\ j = 1,\ldots,n_i,\ i = 1,\ldots,t,\ \ A_i \sim NID(0,\sigma_A^2),\ \ \epsilon_{ij} \sim NID(0,\sigma^2)$

3. Note the difference in expected value, variance and covariance of Y_{ij} between the fixed and random effect one-way ANOVA model:

$$\text{Fixed: } \mathrm{E}(Y_{ij}) = \mu + \alpha_i, \ \ \mathrm{Var}(Y_{ij}) = \sigma^2, \ \ \mathrm{Cov}(Y_{ij}, Y_{ik}) = 0$$

$$\text{Random: } \mathrm{E}(Y_{ij}) = \mu, \ \ \mathrm{Var}(Y_{ij}) = \sigma_a^2 + \sigma^2, \ \ \mathrm{Cov}(Y_{ij}, Y_{ik}) = \sigma_a^2$$

4. To test for treatment effect, the same F-test with test statistic $\frac{SSA/(t-1)}{RSS/(n-t)}$ is applied for both fixed and random effect model. The hypotheses are

$$\text{Fixed: } H_0: \ \alpha_i = 0 \quad \text{vs} \quad H_1: \ \text{not all } \alpha_i = 0$$

$$\text{Random: } H_0: \ \sigma_a^2 = 0 \quad \text{vs} \quad H_1: \ \sigma_a^2 \neq 0$$

5. Since $\mathrm{E}(\bar{Y}_{\bullet\bullet}) = \mu$ and so $\hat{\mu} = \bar{Y}_{\bullet\bullet}$. Its variance estimate is $\widehat{\mathrm{V}}\mathrm{ar}(\bar{Y}_{\bullet\bullet}) = \frac{1}{n}\sigma^2 + \left(\frac{\sum_{i=1}^t n_i^2}{n^2}\right)\sigma_A^2 = \frac{SSA/(t-1)}{n}$. The CI is $\hat{\mu} \pm t_{t-1}(0.975)\sqrt{\frac{SSA/(t-1)}{n}}$.

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last adjustments: May 16, 2020 by JC