STAT 3011/3911

Semester 1	Time Series Analysis	2008

Week 7

Tutorial Exercise.

1. Suppose a random variable X_t at time $t > 0$ has a Gamma distribution with density

$$f(x) = \frac{1}{\Gamma(\alpha)} \beta^{-\alpha} x^{\alpha-1} e^{-x/\beta}.$$

   Determine a transformation of X_t which will approximately stabilize the variance with respect to t in the following cases.

   (a) $\alpha = t \quad \beta = 1$.

   (b) $\alpha =$constant, $\quad \beta = t$.

   (c) $\alpha = \beta = t$.

2. In case (b) of question 1 with $\alpha = 1$, how would a realisation of X_t be simulated, for a given t, by using an observation from a $U(0,1)$ distribution? (Recall that if X has distribution function F then $Y = F(X) \sim U(0,1)$.)

3. Suppose $X_t = \alpha + \beta t + s_t + \epsilon_t$, where $s_t = s_{t+d}$, and $E(\epsilon_t) = 0$, that is, X_t has a linear trend and a seasonal component of period d. Show that differencing at lag d removes both the trend and seasonal component(i.e. consider $Y_t = \nabla_d X_t = X_{t+d} - X_t$.)

4. Given the following 16 observations from a time series calculate the autocorrelation coefficients r_1 and r_2.

   1.6, 0.8, 1.2, 0.5, 0.9, 1.1, 1.1, 0.6, 1.5, 0.8, 0.9, 1.2, 0.5, 1.3, 0.8, 1.2

5. If a time series has a dominant linear trend then the sample autocorrelation coefficients will all be close to 1. To illustrate this consider

$$x_t = \beta t, \quad t = -N, \ldots, N,$$

   where β is a fixed constant. For each N, $\bar{x} = 0$. Show that $r_k \to 1$ as $N \to \infty$ for each fixed k.