



- (a)  $z + w$       (b)  $z - w$       (c)  $|z|$       (d)  $\bar{w}$       (e)  $zw$       (f)  $\frac{z}{w}$

### Questions to do in class

4. Locate each of the following sets on the real number line and then express each as an interval or as a union of intervals:

- (a)  $\{x \in \mathbb{R} \mid 2 \leq x \leq 4\}$       (c)  $[2, 5] \cap (3, 6]$   
 (b)  $\{x \in \mathbb{R} \mid -1 < x \leq 1 \text{ or } x \geq 5\}$       (d)  $\{x \in \mathbb{R} \mid |x - 1| > 2\}$

5. Express the following complex numbers in cartesian form:

- (a)  $(2 + 3i) + (5 - 6i)$       (e)  $(2 + 3i)(5 - 6i)$   
 (b)  $(2 + 3i) - (5 - 6i)$       (f)  $\frac{2 + 3i}{5 - 6i}$   
 (c)  $(1 + i)(1 - i)$       (g)  $\frac{1}{i} - \frac{3i}{1 - i}$   
 (d)  $\frac{1 + i}{1 - i}$       (h)  $i^{123} - 4i^9 - 4i$

6. Solve the following equations over  $\mathbb{C}$ :

- (a)  $z^2 + 3z + 2 = 0$       (c)  $3z^2 - 4z + 4 = 0$   
 (b)  $z^2 + z + 1 = 0$       (d)  $z^4 = 16$

7. Sketch the following regions in the complex plane:

- (a)  $\{z \in \mathbb{C} \mid |z| \leq 3\}$       (d)  $\{z \in \mathbb{C} \mid \text{Im } z \geq -1\}$   
 (b)  $\{z \in \mathbb{C} \mid |z + i| > 2\}$       (e)  $\{z \in \mathbb{C} \mid |z - i| \leq |z - 1|\}$   
 (c)  $\{z \in \mathbb{C} \mid \text{Re } z < -1\}$

8. (Suitable for group discussion.) Are each of these statements true or false? Explain your answer.

- (a) The square of an imaginary number is always real.  
 (b) It does not make sense to write  $|z| > |w|$  when  $z$  and  $w$  are complex numbers because the complex numbers are not ordered.  
 (c) Real numbers cannot be graphed on the complex plane.  
 (d) When a real number is divided by a complex number the answer can never be real.

### Questions for further practice

9. On the complex plane, graph the solutions to the equations in Question 6. What pattern do you notice in each pair of solutions? Explain why this pattern occurs.

10. Locate the following sets, which are given in interval notation, on the real number line. Rewrite each set using  $\{\dots \mid \dots\}$  notation.

(a)  $[-7, -1)$

(b)  $(-\infty, -2] \cup [2, \infty)$

(c)  $(2, 3) \cup [5, 6]$

11. Use a Venn diagram with three sets  $A$ ,  $B$  and  $C$  to show the following:

(a)  $A \cup B \cup C$

(b)  $A \cap B \cap C$

(c)  $(A \cup B) \cap C$

(d)  $A \cup (B \cap C)$

(e)  $(A \cup B \cup C) \setminus (A \cap B \cap C)$

(f)  $(A \setminus (B \cup C)) \cup (B \setminus (A \cup C)) \cup (C \setminus (A \cup B))$

### Short answers to selected exercises

Full solutions to all questions can be downloaded at the end of the week.

1. The answers are as follows:

(a)  $\{1, 2, 3, 4, 5, 6\}$

(c)  $\{1, 2\}$

(e)  $\{1, 2, 5, 6\}$

(b)  $\{3, 4\}$

(d)  $\{5, 6\}$

2.  $X = \{-2, -2, 0, 1, 2\}$ .

(a) True

(b) False

(c) False

(d) False

3. (a)  $-2 + 2i$

(c)  $\sqrt{5}$

(e)  $-5 + 10i$

(b)  $6 - 4i$

(d)  $-4 - 3i$

(f)  $-(11 + 2i)/25$

4. (a)  $[2, 4]$

(c)  $(3, 5]$

(b)  $(-1, 1] \cup [5, \infty)$

(d)  $(-\infty, -1) \cup (3, \infty)$

5. (a)  $7 - 3i$

(c) 2

(e)  $28 + 3i$

(g)  $(3 - 5i)/2$

(b)  $-3 + 9i$

(d)  $i$

(f)  $(-8 + 27i)/61$

(h)  $-9i$

6. (a)  $-1, -2$

(b)  $(-1 \pm i\sqrt{3})/2$

(c)  $(2 \pm 2i\sqrt{2})/3$

(d)  $\pm 2, \pm 2i$

9. All roots are either real or occur in complex conjugate pairs.

10. (a)  $[-7, -1) = \{x \in \mathbb{R} \mid -7 \leq x < -1\}$

(b)  $(-\infty, -2] \cup [2, \infty) = \{x \in \mathbb{R} \mid x \leq -2 \text{ or } x \geq 2\}$

(c)  $(2, 3) \cup [5, 6] = \{x \in \mathbb{R} \mid 2 < x < 3 \text{ or } 5 \leq x \leq 6\}$