

Practice Questions for Quiz 2, MATH1001 Summer School

Quiz 2 is intended to assess lecture material from weeks 3 and 4 (corresponding tutorials are 5 to 8). This material can be found in written form in the course notes, chapters 5 to 8. The second quiz will be held in tutorials on **Tuesday week 5 (7th February) in Lectures.**

Quiz 2 will be in the same format as the first quiz, 7 questions with some questions with multiple parts. The quiz will be out of 10.

1. Compute all of the first and second order partial derivatives for $g(x, y) = 6x^2y + y^2 \ln(x)$.

$$\begin{aligned} g_x &= 12xy + \frac{y^2}{x} & g_y &= 6x^2 + 2y \ln(x) \\ g_{xx} &= 12y - \frac{y^2}{x^2} & g_{yy} &= 2 \ln(x) \\ g_{xy} &= 12x + \frac{2y}{x} & g_{yx} &= 12x + \frac{2y}{x} \end{aligned}$$

2. Compute all the first order partial derivatives for $f(x, y) = \ln(\sin(x^2y^3))$.

$$f_x = \frac{2xy^3 \cos(x^2y^3)}{\sin(x^2y^3)} \quad f_y = \frac{3x^2y^2 \cos(x^2y^3)}{\sin(x^2y^3)}$$

3. Calculate the partial derivatives $f_x, f_y, f_{xx}, f_{yy}, f_{xy}$ and f_{yx} of $f(x, y) = e^{\cos(x+2y)}$ at the point $(\frac{\pi}{2}, 0)$.

Answers respectively are:
-1, -2, 1, 4, 2, 2

4. Find the value of $f(x, y) = 4x^2 + y^2 + 8x - 8y$ at its critical point, and compute its discriminant.

$$-20, D = 16$$

5. Compute the equation of the tangent line to the level curve $f(x, y) = e^2$ at the point $(0, 2)$ where $f(x, y) = x^2 + e^{y-2x}$.

$$y = 2x + 2$$

6. Let $f(x, y) = \sin(x^2 - e^y)$. Find an expression for the differential df of f in terms of dx and dy at the point $(1, 0)$.

$$df = 2 dx - dy$$

7. Find all the critical points for $f(x, y) = x^3 + y^3 - 3xy + 1$.

$$(0, 0), (1, 1).$$

8. Determine the nature of all critical points of $f(x, y) = x^3 + y^3 - 3xy + 1$
- saddle at $(0, 0)$, local min at $(1, 1)$.
9. Find the equation of the tangent plane to the surface $z = x^2 + y^2 + 5xy$ at the point $(2, 1)$.
- $9x + 12y - z = 15$
10. Let $f(x, y) = \cos(2x) + e^y$, and let $x = \cos(st)$, $y = 2 \sin s$. Calculate $\frac{\partial f}{\partial t}$ for $s = 1$, $t = \frac{\pi}{4}$.
- $\sqrt{2} \sin(\sqrt{2})$
11. Let $f(x, y) = x^2 + 3y^2$, and let $x = e^t$, $y = -t$. Calculate $\frac{df}{dt}$ for $t = 1$.
- $2e^2 + 6$