Solutions and Markscheme for
Summer School 2011 Examination
Extended Response Section

1. (a) 1 mark for each correct vector, no half marks awarded.

(b) 1 mark for the correct cross product (no half marks). 1/2 a mark each for each correct dot product required for the equation.

(c) Either

- found the right vector for a point on the line (1 mark)
- found the right vector parallel to the line (1 mark)
- wrote down the right cartesian eqn (1 mark)

or

- Tried solving equations simultaneously. Here a mark was only awarded if the process they used is correct (i.e. a proper substitution or row reduction of a matrix) (1 mark)
- correct solution to paramatric scalar form (1 mark), half a mark otherwise if student made a concerted effort, such as getting to the point where they found a parameter.
- correct Cartesian eqn from previous eqn (1 mark), half a mark if a good effort was made.

or

- 1 mark if student indicated some knowledge of what an equation of a line is, using a vector to denote a point on the line and another to denote the vector the line is parallel to, e.g.:

$$\mathbf{x} = \mathbf{x}_0 + t\mathbf{v}$$

2. (a) Students received one mark for writing one of the following (no half marks were awarded):

- 1 mark if student writes an equation like $aM\mathbf{u} + bM\mathbf{v}$ and applies the relations $M\mathbf{u} = -\mathbf{u}$ etc correctly to obtain $-a\mathbf{u} + b\mathbf{v}$. It was also acceptable for students to use non zero values for a and b.
  or
- 1 mark awarded if students write a statement like

$$a\mathbf{u} + b\mathbf{v} = 0 \implies \mathbf{u} = k\mathbf{v}$$

  Rmk: $a\mathbf{u} = -b\mathbf{v}$ is not accepted.
  or
- 1 mark if they say "as eigenvalues are not equal $\mathbf{u}$ and $\mathbf{v}$ cannot be scalar multiples of each other".

(b) Either

- 1/2 mark for each eigenvalue written down in (i), and 1 for writing $\lambda = 1$ for (ii). Any statement where you can clearly tell the student meant to write -1 or 1 (and in the case of part (ii) 1) was accepted.
  or

- 1 mark for the whole question if students define eigenvalue λ using the eqn $M\mathbf{v} = \lambda\mathbf{v}$.

AND if marks have not been awarded in each part for reasons above:

- 1/2 a mark for writing each of (i)$\det(M - \lambda I) = 0$ and (ii) $\det(M^2 - \lambda I) = 0$.

(c) Either

- $M = PDP^{-1}$ nets them 1 mark. Further explaining why M is invertible by calculating M^2, or writing the inverse of M by expanding $(PDP^{-1})^{-1}$, or saying "since M is the product of invertible matrices", nets them the other mark.
  or
- (1 mark) If they say $\det(M) \neq 0$ implies M^{-1} exists or invoke a characteristic eqn like statement such as $\det(M - 0I)$.
  or
- (1 mark) If they say M is its own inverse or illustrate a statement close to this. Fully explaining why M is its own inverse gets them the other mark.

(d) Either

- (1 mark) Expressing vector relations with M correctly by substituing the vectors given into: $M\mathbf{u} = -\mathbf{u}$, $M\mathbf{v} = \mathbf{v}$, $M\mathbf{w} = -\mathbf{w}$. If $\mathbf{u}$ and only one other vector was substitued correctly half a mark was awarded.
- (1 mark) Let $M = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and multiplying out the LHS of the above relations correcty. If two out of 3 multiplications were correct half a mark was awarded.
- (1 mark) Realising at some point they have to solve simultaneous eqns.
- (2 marks) Solving the simultaneous eqns correctly. 1 mark awarded for a good effort. If insufficient working or final solutions were wrong 1 mark was deducted.

or

- (1 mark) Expressing vector relations with M correctly by writing $MP = MD$ (or equivalent), for the correct P and D. If only two of the column pairs in P and D were correct half a mark was awarded.
- (1 mark) Realising P is invertible.
- (2 marks) Solving for P^{-1}. 1 mark was awarded if sufficient competency was demonstrated.
- (1 mark) Multiplying out PDP^{-1}. half a mark was awarded if sufficient competency was demonstrated.

or

- 1 mark was awarded if students row reduced a suitable 3×3 matrix to the identity, such as a matrix made up of the eigenvectors.

3. (a) (1 mark) for all vectors correct, ($\frac{1}{2}$ marks) for at least one correct.

(b) ($\frac{1}{2}$ marks) for quoting the orthogonality criterion and ($\frac{1}{2}$ marks) for attempting to use it, (1 mark) for correct answer.

- (1 mark) for an incomplete geometric argument (using isoceles triangle, parallelogram or rhombus properties without proof)
- (0 marks) for explaining the hint.

(c) ($\frac{1}{2}$ marks) for an explation of why we choose $\mathbf{a}$, ($\frac{1}{2}$ marks) for using part (b) to conclude that $\mathcal{L}_1$ is parallel to $\mathbf{b} + \mathbf{c}$.

(d) (1 mark) for expansion of equation, with ($\frac{1}{2}$ marks) awarded for demonstration of some competency. (1 mark) for the conclusion that $t = s = 1$ and (1 mark) for identifying that the position vector of the point of intersection of $\mathcal{L}_1$ and $\mathcal{L}_2$ is the vector $\overrightarrow{OH}$.