THE UNIVERSITY OF SYDNEY

MATH1002 LINEAR ALGEBRA

Summer School **Assignment** 2012

This assignment is due by **10am on Thursday 9 February (week 6)**. It should be handed in with a **signed cover sheet including your name and SID stapled to the front** at the beginning of the lecture. This assignment is worth 15% of your final mark.

1. Let

$$\mathbf{v}_1 = 2\mathbf{i} + \mathbf{j} + 4\mathbf{k},$$
$$\mathbf{v}_2 = 4\mathbf{i} - \mathbf{j} + 5\mathbf{k}$$

be vectors, and suppose the line $\mathcal{L}_1$ is given by

$$\mathbf{r}_1 = -8\mathbf{i} + 2\mathbf{j} - 9\mathbf{k} + s\mathbf{v}_1, \quad s \in \mathbb{R},$$

and the line $\mathcal{L}_2$ is given by

$$\mathbf{r}_2 = 15\mathbf{i} + 9\mathbf{j} + 7\mathbf{k} + t\mathbf{v}_2, \quad t \in \mathbb{R}.$$

(a) Verify that $\mathcal{L}_1$ and $\mathcal{L}_2$ do not intersect. **[3]**

(b) Find a vector $\mathbf{u}$ that is perpendicular to both $\mathcal{L}_1$ and $\mathcal{L}_2$. **[2]**

(c) Solve the vector equation

$$n\mathbf{u} = \mathbf{r}_1 - \mathbf{r}_2$$

for n, s and t. **[3]**

(d) Find the points on $\mathcal{L}_1$ and $\mathcal{L}_2$ corresponding to the values of s and t you found in part (c), and give a geometrical interpretation of the quantity $|n| \cdot |\mathbf{u}|$. **[2]**

2. For each of the following statements, find either a proof (if true) or a counterexample (if false):

(a) A system of two linear equations in three variables has a 1-parameter family of solutions. **[2]**

(b) Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be $n \times n$ matrices, and suppose $a_{ij} = b_{ji}$ for all i, j. Then $\det A = \det B$. **[2]**

(c) For any matrix A, if $A^2 \neq 0$ then $A^3 \neq 0$. **[2]**

(d) Let A be a 2×2 matrix, and let $\mathrm{tr}(A)$ (the *trace* of A) be the sum of the diagonal entries. Show that

$$A^2 - \mathrm{tr}(A)A + \det(A)I_2 = 0_{2\times 2}.$$

[2]

(e) Define

$$T_n(A) = I_n + \frac{1}{1!}A + \frac{1}{2!}A^2 + \ldots + \frac{1}{n!}A^n,$$

and set

$$e^A = \lim_{n\to\infty} T_n(A).$$

Then $e^{A+B} = e^A e^B$ if and only if $AB = BA$. **[2]**