4. (i) 6 (ii) 1 (iii) 6 (iv) 2/3

5. 
\[ \begin{align*} 
\vec{b} \ &= \vec{a} - \vec{b} \\
\vec{b} \ &= \vec{b} - \vec{a} \\
\vec{b} \ &= \vec{a} + \vec{b} 
\end{align*} \]

6. (i) \( \vec{u} - \vec{v} - \vec{w} \) (ii) \( \vec{u} + \vec{v} - \vec{w} \) (iii) \( -\vec{u} - \vec{v} + \vec{w} \)

7. 
\[ \begin{align*} 
8 \\
\theta \\
d \\
6 
\end{align*} \]

By Pythagoras \( d = \sqrt{8^2 + 6^2} = 10 \). If \( \theta \) is the angle to the horizontal then \( \cos \theta = 6/10 \), yielding an angle \( \theta \approx 53^\circ \). Thus the resultant force is 10 newtons in a direction 53° to the horizontal, towards the right.

8. The associative law for addition of vectors says that, for any vectors \( \vec{u}, \vec{v} \) and \( \vec{w} \),
\[ \vec{u} + (\vec{v} + \vec{w}) = (\vec{u} + \vec{v}) + \vec{w}. \]

To verify this, we suppose that the vectors have been lined up so that the point \( P \) is at the tail of \( \vec{u} \), the point \( Q \) is both at the tip of \( \vec{u} \) and at the tail of \( \vec{v} \), the point \( R \) is both at the tip of \( \vec{v} \) and at the tail of \( \vec{w} \), and the point \( S \) is at the tip of \( \vec{w} \). Then
\[ \begin{align*} 
\vec{u} + (\vec{v} + \vec{w}) & = \overrightarrow{PQ} + (\overrightarrow{QR} + \overrightarrow{RS}) \\
& = \overrightarrow{PQ} + \overrightarrow{QS} \\
& = \overrightarrow{PR} + \overrightarrow{RS} \\
& = (\overrightarrow{PQ} + \overrightarrow{QR}) + \overrightarrow{RS} \\
& = (\vec{u} + \vec{v}) + \vec{w}. 
\end{align*} \]
9. Place the vectors \( \mathbf{v} \) and \( \mathbf{w} \) tip-to-tail so that they label two directed edges of a triangle \( ABC \), so that

\[
\mathbf{v} = \overrightarrow{AB}, \quad \mathbf{w} = \overrightarrow{BC}.
\]

Then \( \mathbf{v} + \mathbf{w} = \overrightarrow{AC} \). The shortest distance between two points is a straight line, so that travelling from \( A \) to \( C \) via \( B \) is at least as far as travelling directly from \( A \) to \( C \).

Thus

\[
|\mathbf{v} + \mathbf{w}| = |\overrightarrow{AC}| \leq |\overrightarrow{AB}| + |\overrightarrow{BC}| = |\mathbf{v}| + |\mathbf{w}|,
\]

which verifies the triangle inequality. This becomes equality precisely when \( B \) falls on the direct path joining \( A \) to \( C \), so that the triangle becomes degenerate.

10.*

Observe that

\[
\overrightarrow{DE} = \overrightarrow{DB} + \overrightarrow{BE} = \frac{1}{2} \overrightarrow{AB} + \frac{1}{2} \overrightarrow{BC} = \frac{1}{2} (\overrightarrow{AB} + \overrightarrow{BC}) = \frac{1}{2} \overrightarrow{AC}.
\]

This tells us that the line segment joining \( D \) to \( E \) is parallel to and half the length of the line segment joining \( A \) to \( C \).

11. \( 2a - 3b = -7u + 8v \).

12. \( \overrightarrow{CD} = \mathbf{b} - \mathbf{a} \), \( \overrightarrow{DE} = -\mathbf{a} \), \( \overrightarrow{EF} = -\mathbf{b} \), \( \overrightarrow{FA} = \mathbf{a} - \mathbf{b} \).
13.

We have $|\overrightarrow{AB}| = 20$ and $|\overrightarrow{BC}| = 10$. By the Cosine Rule,

$$|\overrightarrow{AC}| = \sqrt{20^2 + 10^2 - 2(10)(20) \cos 105^\circ} \approx 25.$$ 

By the Sine Rule,

$$\sin(30^\circ - \alpha) = \frac{10 \sin 105^\circ}{|\overrightarrow{AC}|},$$

from which it follows that

$$30^\circ - \alpha \approx 23^\circ,$$

so that $\alpha \approx 7^\circ$. Hence the final distance and direction of the aircraft from the starting point are approximately 25 km and 7° north of east respectively.

14.*

$$\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BD} = \overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC} = \overrightarrow{AB} + \frac{1}{2}(\overrightarrow{BA} + \overrightarrow{AC})$$

$$= \overrightarrow{AB} + \frac{1}{2}(-\overrightarrow{AB} + \overrightarrow{AC}) = \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{AC} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{AC}).$$

15.* Let $P, Q, R, S$ be the respective midpoints of the edges $AB, BC, CD, DA$ of the quadrilateral $ABCD$. Then, by two applications of Exercise 10, firstly to the triangle $ABC$, and then secondly to the triangle $ADC$,

$$\overrightarrow{PQ} = \frac{1}{2}\overrightarrow{AC} = \overrightarrow{SR},$$

which is sufficient to prove that $PQRS$ is a parallelogram.