4. (i) 6 (ii) 1 (iii) 6 (iv) \(2/3\)

5. 

\[
\begin{align*}
\text{b} & = \text{a} - \text{b} \\
\text{b} & = \text{b} - \text{a} \\
\text{b} & = \text{a} + \text{b}
\end{align*}
\]

6. (i) \(\mathbf{u} - \mathbf{v} - \mathbf{w}\)  (ii) \(\mathbf{u} + \mathbf{v} - \mathbf{w}\)  (iii) \(-\mathbf{u} - \mathbf{v} + \mathbf{w}\)

7. 

By Pythagoras \(d = \sqrt{8^2 + 6^2} = 10\). If \(\theta\) is the angle to the horizontal then \(\cos \theta = 6/10\), yielding an angle \(\theta \approx 53^\circ\). Thus the resultant force is 10 newtons in a direction 53\(^\circ\) to the horizontal, towards the right.

8. The associative law for addition of vectors says that, for any vectors \(\mathbf{u}, \mathbf{v}\) and \(\mathbf{w}\),

\[
\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w} .
\]

To verify this, we suppose that the vectors have been lined up so that the point \(P\) is at the tail of \(\mathbf{u}\), the point \(Q\) is both at the tip of \(\mathbf{u}\) and at the tail of \(\mathbf{v}\), the point \(R\) is both at the tip of \(\mathbf{v}\) and at the tail of \(\mathbf{w}\), and the point \(S\) is at the tip of \(\mathbf{w}\). Then

\[
\begin{align*}
\mathbf{u} + (\mathbf{v} + \mathbf{w}) &= \overrightarrow{PQ} + (\overrightarrow{QR} + \overrightarrow{RS}) = \overrightarrow{PQ} + \overrightarrow{QS} \\
&= \overrightarrow{PS} \\
&= \overrightarrow{PR} + \overrightarrow{RS} = (\overrightarrow{PQ} + \overrightarrow{QR}) + \overrightarrow{RS} = (\mathbf{u} + \mathbf{v}) + \mathbf{w} .
\end{align*}
\]
9. Place the vectors $v$ and $w$ tip-to-tail so that they label two directed edges of a triangle $ABC$, so that
\[ v = \overrightarrow{AB}, \quad w = \overrightarrow{BC}. \]
Then $v + w = \overrightarrow{AC}$. The shortest distance between two points is a straight line, so that travelling from $A$ to $C$ via $B$ is at least as far as travelling directly from $A$ to $C$. Thus
\[ |v + w| = |\overrightarrow{AC}| \leq |\overrightarrow{AB}| + |\overrightarrow{BC}| = |v| + |w|, \]
which verifies the triangle inequality. This becomes equality precisely when $B$ falls on the direct path joining $A$ to $C$, so that the triangle becomes degenerate.

10.*

11. \[ 2a - 3b = -7u + 8v. \]

12. \[ \overrightarrow{CD} = b - a, \quad \overrightarrow{DE} = -a, \quad \overrightarrow{EF} = -b, \quad \overrightarrow{FA} = a - b. \]
13. (Homework)

14.∗ (Homework)

15.∗ Let $P$, $Q$, $R$, $S$ be the respective midpoints of the edges $AB$, $BC$, $CD$, $DA$ of the quadrilateral $ABCD$. Then, by two applications of Exercise 10, firstly to the triangle $ABC$, and then secondly to the triangle $ADC$,

$$\overrightarrow{PQ} = \frac{1}{2}\overrightarrow{AC} = \overrightarrow{SR},$$

which is sufficient to prove that $PQRS$ is a parallelogram.