Assignment 1

This assignment is due by **11:59pm Monday 22nd of January 2018**, via Turnitin. A PDF copy of your answers must be uploaded in the Learning Management System (Canvas) at https://canvas.sydney.edu.au/. Please submit only a PDF document (scan or convert other formats). It should include your name and SID; your tutorial time, day, room and Tutor's name. It is your responsibility to preview each page of your assignment after uploading to ensure each page is included in correct order and is legible (not sideways or upside down) before confirming your submission, and then to check your submission receipt. The School of Mathematics and Statistics encourages some collaboration between students when working on problems, but students must write up and submit their own version of the solutions.

This assignment is worth 5% of your final assessment for this course. Your answers should be well written, neat, thoughtful, mathematically concise, and a pleasure to read. Please cite any resources used and show all working. Present your arguments clearly using words of explanation and diagrams where relevant. After all, mathematics is about communicating your ideas. This is a worthwhile skill which takes time and effort to master. The marker will give you feedback and allocate an overall letter grade and mark to your assignment using the following criteria:

Mark	Grade	Criterion
10	A+	Outstanding and scholarly work, answering all parts correctly, with clear accurate explanations and all relevant diagrams and working. There are at most only minor or trivial errors or omissions.
9	A	Very good work, making excellent progress on at least 4 of the 5 parts and good progress on the remaining parts, but with one or two substantial errors, misunderstandings or omissions throughout the assignment.
7	B	Good work, making good progress on 3 parts and some progress on the remaining parts, but making more than two distinct substantial errors, misunderstandings or omissions throughout the assignment.
6	C	A reasonable attempt, making substantial progress on only 3 of the 5 parts.
4	D	Some attempt, with substantial progress made on only 2 parts.
2	E	Some attempt, with substantial progress made on only 1 part.
0	F	No credit awarded.

1. (a) Find the Lower L_5 and Upper U_5 Riemann Sums for the definite integral

$$\int_{-3}^{2} x^2 dx$$

   (Hint: Not uniformly increasing/decreasing.)

   (b) How many subintervals in a Riemann sum will guarantee the approximate area under $f(x) = x^2$ on the interval $[-3, 2]$ is accurate to 2 d.p.?
   (Hint: Error $\leq U_N - L_N$)

2. Question 1b) from the 2017 Summer School Exam.
   (a) Sketch the area bounded by the curves $y = e^{1+2x}$, $y = e^{1-x}$, $x = -2$ and $x = 1$.

   (b) Where do the curves intersect?

   (c) Find the bounded area. (Hint: Two separate regions, two integrals.)

3. (a) Sketch the area in the xy-plane bounded by the curves $y = x^2 - 2x$ and $y = x$.
   (Hint: Find the points of intersection, these will be your endpoints.)

   (b) Rotate about the line $y = 4$ and determine the volume of the solid using the disk method. (Hint: Big disk-Little disk)

4. Evaluate the following integrals by making a substitution:
   (a) $\int \frac{dx}{1+e^x}$. Hint: Make a substitution and then "add 0".

   (b) The same integral as in part a). This time however, begin by "multiplying by 1", and then making a substitution.

   (c) Using the log laws and other tricks. Explain why these two answers are equivalent.

5. Question 1a) from the 2017 Summer School Exam.
   (a) Evaluate the following definite integral

$$\int_{1}^{81} \frac{1}{x^2} dx$$

   (b) Using sigma notation, write down expressions for the upper Riemann sum U_N and the lower Riemann sum L_N for the definite integral of part (i) using $N = 80$ equally spaced subintervals.

   (c) Use parts (i) and (ii) or otherwise to derive the following inequalities:

$$\frac{6481}{6561} \leq \sum_{k=1}^{81} \frac{1}{k^2} \leq \frac{161}{81}$$

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