The University of Sydney
School of Mathematics and Statistics

Assignment 2

MATH1003: Integral Calculus and Modelling — Summer School 2018

Web Page: http://sydney.edu.au/science/maths/u/UG/SS/SS1003/
Lecturers: Alexander Majchrowski

This assignment is due by **11:59pm Monday 5th of February 2018**, via Turnitin. A PDF copy of your answers must be uploaded in the Learning Management System (Canvas) at https://canvas.sydney.edu.au. Please submit only a PDF document (scan or convert other formats). It should include your name and SID; your tutorial time, day, room and Tutor's name. It is your responsibility to preview each page of your assignment after uploading to ensure each page is included in correct order and is legible (not sideways or upside down) before confirming your submission, and then to check your submission receipt. The School of Mathematics and Statistics encourages some collaboration between students when working on problems, but students must write up and submit their own version of the solutions.

This assignment is worth 5% of your final assessment for this course. Your answers should be well written, neat, thoughtful, mathematically concise, and a pleasure to read. Please cite any resources used and show all working. Present your arguments clearly using words of explanation and diagrams where relevant. After all, mathematics is about communicating your ideas. This is a worthwhile skill which takes time and effort to master. The marker will give you feedback and allocate an overall letter grade and mark to your assignment using the following criteria:

Mark	Grade	Criterion
10	A+	Outstanding and scholarly work, answering all parts correctly, with clear accurate explanations and all relevant diagrams and working. There are at most only minor or trivial errors or omissions.
9	A	Very good work, making excellent progress on at least 4 of the 5 parts and good progress on the remaining parts, but with one or two substantial errors, misunderstandings or omissions throughout the assignment.
7	B	Good work, making good progress on 3 parts and some progress on the remaining parts, but making more than two distinct substantial errors, misunderstandings or omissions throughout the assignment.
6	C	A reasonable attempt, making substantial progress on only 3 of the 5 parts.
4	D	Some attempt, with substantial progress made on only 2 parts.
2	E	Some attempt, with substantial progress made on only 1 part.
0	F	No credit awarded.

Copyright © 2018 The University of Sydney

1. In lectures it was shown that for

$$G(x) = \int_0^{h(x)} f(t)dt$$

we have $\frac{d}{dx}G(x) = F'(h(x))h'(x)$ where $F(x) = \int_0^x f(t)dt$.

(a) Using the above, find a formula for $\frac{d}{dx}G(x)$, where

$$G(x) = \int_{g(x)}^{0} f(t)dt.$$

(b) Hence find a formula for $\frac{d}{dx}G(x)$, where

$$G(x) = \int_{g(x)}^{h(x)} f(t)dt.$$

(c) Using your new formula find $\frac{d}{dx}G(x)$, where

$$G(x) = \int_{cos(x)}^{sin(x)} \sqrt{1-t^2}dt$$

2. Find $\frac{dy}{dx}$ for the following:

(a) $y = \dfrac{(ln(x))^{x+1}}{2^{2x^2+3x+1}}$

(b) $y = (ln(x))^{\sin(x)+\cos(x)}$

(c) $y = \log_{10} x^3 - \log_2 \sin(x)$

3. (a) Sketch the direction field for the following differential equation:

$$\frac{dy}{dx} = y - x^2.$$

(b) Sketch three different solutions. One with no turning point, another with one turning points and the last with two turning points.

4. Find a General Solution and a Particular Solution to the following:

(a) $\dfrac{dy}{dx}\ln(x) - \dfrac{y}{x} = 0, \quad y(e) = 1$

(b) $e^x\dfrac{dy}{dx} - xy = 0, \quad y(0) = \ln(5)$

5. Find a reduction formula for

$$I_n = \int \frac{x^n}{\sqrt{ax+b}}dx.$$

Hint: Use the fact that $\sqrt{y} = \frac{y}{\sqrt{y}}$.