Assumed Knowledge: Sketching curves of simple functions. Integrals of simple functions such as $x^n$ (including $1/x$), $\sin x$, $\cos x$, $e^x$.

Objectives:

(3a) To be able to interpret an area or volume as the limit of an appropriate Riemann sum.

(3b) To be able to use elementary slices to construct Riemann sums for areas and volumes.

(3c) To be able to use elementary shells to construct Riemann sums for volumes.

(3d) To understand the substitution formula and be able to use simple substitutions to evaluate definite integrals.

Exercises:

1. Evaluate the following definite integrals using a substitution.
   
   (a) $\int_{0}^{1/2} \frac{x}{\sqrt{1-x^2}} \, dx$.
   
   (b) $\int_{0}^{\pi/2} \sin^3 x \cos^4 x \, dx$.
   
   (c) $\int_{0}^{1} (2x+1)(x^2+x+1)^3 \, dx$.

2. Sketch the region of the $xy$-plane bounded by the $x$-axis, the line $x = 2$, and the graph of $y = x$. This region is rotated about the line $x = 4$. Sketch an elementary disk that you would use to construct the volume. Hence construct an integral for the volume and evaluate this integral to find the volume.

3. Evaluate the following definite integrals using the substitution $u = \sec x$.
   
   (a) $\int_{0}^{\pi/4} \tan x \sec^3 x \, dx$.
   
   (b) $\int_{0}^{\pi/3} \sec^5 x \tan^3 x \, dx$. (Hint: Use a trigonometric identity.)
Brief answers to selected exercises:

1. (a) \(-\sqrt{\frac{3}{2}} + 1\).
   (b) \(\frac{2}{35}\)
   (c) 20

2. \(\frac{32\pi}{3}\)

3. (a) \(\frac{2\sqrt{3}}{3} - \frac{1}{3}\)
   (b) \(\frac{418}{35}\)