Assumed Knowledge: Solving simultaneous linear equations.

Objectives:

(12a) To be able to rewrite two coupled first-order differential equations as a single second-order differential equation.

(12b) To be able to sketch the solutions of second-order differential equations with constant coefficients.

Exercises:

1. Find the general solution of the following system of equations:

\[ \frac{dx}{dt} = x + y, \quad \frac{dy}{dt} = 3x - y . \]

2. Find the general solution of the pair of differential equations

\[ \frac{dx}{dt} = 5x - 3y, \quad \frac{dy}{dt} = 2y , \]

by first solving the second equation and then substituting into the first. (There are two equations, so you should have two arbitrary constants of integration at the end.)

Find the particular solution satisfying the initial conditions \( x = 1, y = 2 \) when \( t = 0 \).

3. (a) For each of the following systems of differential equations for \( x(t) \) and \( y(t) \), find an equivalent second-order differential equation. (You do not need to find any solutions.)

(i) \( x' = y + 4x, \quad y' = 6x \).

(ii) \( x' = y, \quad y' = x + 5y \).

(b) Eliminate \( y(t) \) from the following system to obtain a nonlinear second-order equation for \( x(t) \). (You do not need to find any solutions.)

\[ x' = y, \quad y' = x - xy + 3t . \]
Brief answers to selected exercises:

1. $x = Ae^{2t} + Be^{-2t}$ and $y = Ae^{2t} - 3Be^{-2t}$

2. $x = 2e^{2t} - e^{5t}$ and $y = 2e^{2t}$

3. (a) (i) $x'' - 4x' - 6x = 0$
    (ii) $y'' - 5y' - y = 0$

   (b) $x'' + xx' - x = 3t$