This tutorial covers material from the lectures in Day 4.

1. Evaluate the following integrals by using integration by parts.
   (See the back of this page if you need to check the general formula.)
   (a) \( \int_0^{1/2} xe^{2x} \, dx \).
   (b) \( \int_0^{\pi/4} \theta \sin 4\theta \, d\theta \).
   (c) \( \int_1^2 t^2 \ln t \, dt \).

2. Define \( \text{Si}(x) \) as \( \text{Si}(x) = \int_0^x f(t) \, dt \), where \( f(t) = \begin{cases} \sin t & t \neq 0 \\ t & t = 0 \end{cases} \).

   This function is called the \textit{sine-integral}, and is useful in optics.

   This is the graph of \( f(t) \).

   (a) What is \( \text{Si}'(x) \)?
   (b) What is the value of \( \text{Si}(0) \)?
   (c) For \( 0 \leq x \leq 3\pi \), use the graph of \( f(t) \) to determine the values of \( x \) for which \( \text{Si}(x) \) is increasing, and the values of \( x \) for which it is decreasing.
   (d) For which values of \( x \) between 0 and \( 3\pi \) does \( \text{Si}(x) \) have stationary points?
   (e) Use the graph of \( f(t) \) to estimate \( \text{Si}(\pi) \), \( \text{Si}(2\pi) \) and \( \text{Si}(3\pi) \).
   (f) Use the graph of \( f(t) \) to determine values of \( x \) at which \( \text{Si} \) has points of inflection.
   (g) Sketch the graph of \( \text{Si} \) for \( 0 \leq x \leq 3\pi \). Indicate clearly the vertical and horizontal scales on your graph. Be as accurate as possible. You may wish to use the grid overleaf as a guide.
3. Establish the following reduction formula. (Hint: Write the integrand as $u(x)v'(x)$ where $u = \cos^{n-1} x$.)

$$\int \cos^n x \, dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x \, dx.$$  

Use this formula to find $\int \cos^2 x \, dx$ and $\int \cos^4 x \, dx$.

4. (a) Let $I_n = \int x (\ln x)^n \, dx$. Use integration by parts to establish the reduction formula

$$I_n = \frac{1}{2} x^2 (\ln x)^n - \frac{n}{2} I_{n-1}.$$  

(b) Starting with $I_0 = \int x \, dx = \frac{1}{2} x^2 + C$, use the reduction formula from part (a) to find $I_2$.

The **integration by parts** formula is

$$\int_a^b u(x)v'(x) \, dx = [u(x)v(x)]_a^b - \int_a^b u'(x)v(x) \, dx.$$

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