This tutorial covers material from the lectures in Day 8.

1. Use partial fractions to express each of the following as a sum:
   \[ \frac{1}{(x+1)(x-2)} \] (a)\hspace{1cm} \[ \frac{x}{x-5} \] (b)

2. Evaluate the integrals:
   \[ \int \frac{1}{(x+1)(x-2)} \, dx \] (a)\hspace{1cm} \[ \int \frac{x}{x-5} \, dx \] (b)

3. A trout farmer believes that, if she doesn’t remove any fish from a certain tank, the population of fish is modelled by the differential equation
   \[ \frac{dP}{dt} = 2P - 0.01P^2, \]
   where \( P \) is the number of fish after \( t \) years.
   (a) Suppose the farmer initially stocks the tank with 50 fish.
      (i) What is the maximum number of fish the tank can support?
      (ii) Find \( P \) for which the population is increasing most rapidly.
      (iii) Find an explicit formula for \( P \) in terms of \( t \).
      (iv) How long will it be before there are 199 fish in the tank?
      (v) Sketch a graph of \( y = P(t) \).
   (b) Now suppose that the farmer initially stocks the tank with 250 fish. Find the solution \( P(t) \), and sketch its graph.
   (c) If the trout farmer were to remove 75 fish from the tank each year, then the differential equation modelling population size would be
      \[ \frac{dP}{dt} = 2P - 0.01P^2 - 75. \]
      (i) Find a solution to this differential equation, supposing that the farmer initially stocks the tank with 60 fish.
      (ii) According to this model, what is the size of the fish population in the long-term?
      (iii) What should the farmer expect to happen if she initially stocks the tank with 50 fish?
4. (a) Use the geometry of the above right-angled triangle in which \( \tan(x/2) = t \) to show that

(i) \( \sin \frac{x}{2} = \frac{t}{\sqrt{1 + t^2}} \)  
(ii) \( \cos \frac{x}{2} = \frac{1}{\sqrt{1 + t^2}} \)

(b) Hence show that

(i) \( \cos x = \frac{1 - t^2}{1 + t^2} \)  
(ii) \( \sin x = \frac{2t}{1 + t^2} \)

(c) If \( t = \tan \frac{x}{2} \), show that \( \frac{dx}{dt} = \frac{2}{1 + t^2} \).

(d) Use the substitution \( t = \tan \frac{x}{2} \) to evaluate

\[ \int \frac{5}{4 \sin x + 3 \cos x} \, dx. \]