1. Use the notation of set theory to describe:
   (i) The set of all odd integers between 2 and 10.
   (ii) The set of all odd integers between 2 and 200.
   (iii) The set of all odd integers.
   (iv) The set of integers divisible by 4.
   For this exercise, do not use the dots notation (like \ldots).

2. Which of the following statements are true.
   (i) \{2, 4\} \subseteq \{1, 2, 3, 4, 5, 6\}.
   (ii) \{2\} \subseteq \{1, 2, 3, 4, 5, 6\}.
   (iii) 2 \subseteq \{1, 2, 3, 4, 5, 6\}.
   (iv) 2 \in \{1, 2, 3, 4, 5, 6\}.
   (v) \{2\} \in \{1, 2, 3, 4, 5, 6\}.
   Give reasons for your answers.

3. Let \(A = \{1, 2, 3, \{2\}, \{2, 3\}, 4\}\). Which of the following statements are true?
   (i) \{2\} \in A.
   (ii) \{\{2\}\} \subseteq A
   (iii) \{2, \{2\}\} \subseteq A.
   (iv) \{2, \{3\}\} \subseteq A.
   (v) \{2, 3\} \in A.
   (vi) \{3, \{2, 3\}\} \subseteq A.

4. Write out the following sets, where \(A = \{a, b, c, \{a, d\}\}\):
   (i) \(A \cup \{b, d, e\}\).
   (ii) \(A \cap \{b, d, e\}\).
   (iii) \(A \setminus \{a, b\}\).
   (iv) \(A \setminus \{c, d\}\).
   (v) \(A \setminus \{a, \{a, d\}\}\).
   (vi) \(A \setminus \{a, \{a, d\}\}\).

   Then write down the sizes of each of the sets.

5. List the elements in each of the six sets \(P, Q, P \cup Q, P \cap Q, P \setminus Q\) and \(Q \setminus P\), where
   \[P = \{x \mid x \in \mathbb{Z} \text{ and } 4 \leq x \leq 10\},
   Q = \{y \mid y \in \mathbb{Z} \text{ and } \frac{y}{2} \in \mathbb{Z} \text{ and } 0 \leq y^2 \leq 50\}\].

6. Let \(A = \{a, b, c, d\}\). Write down all the subsets of \(A\). How many are there?

7. If \(A\) and \(B\) are subsets of a set \(X\), prove that
   \[X \setminus (A \cap B) = (X \setminus A) \cup (X \setminus B)\].
Problem Set 2

1. Let \( A = \{a, b, c, \{b\}\} \). Determine which of the following statements are true?
   (i) \( \{b\} \in A \);
   (ii) \( \{b\} \subseteq A \);
   (iii) \( \{\{b\}\} \subseteq A \);
   (iv) \( \{a\} \in A \).

2. Let \( A = \{x \mid x \in \mathbb{Z}, -3 < x < 2\} \) and \( B = \{x^2 + 1 \mid x \in \mathbb{Z}, -3 < x < 2\} \).
   (i) Write down the elements of \( A \) and \( B \).
   (ii) Find \( A \cup B \), \( A \cap B \) and \( A \setminus B \).
   (iii) Find \( |A \cup B| \), \( |A \cap B| \) and \( |A \setminus B| \).

3. Let \( A = \{x \in \mathbb{N} \mid 1 \leq x^2 \leq 30\} \) and \( B = \{x \in \mathbb{Z} \mid x = 2y \text{ for some } y \in \mathbb{Z} \text{ and } x^2 < 50\} \).
   (i) List the elements of \( A \) and \( B \).
   (ii) Find \( A \cup B \), \( A \cap B \), \( A \setminus B \), \( B \setminus A \).
   (iii) Find \( |A \cup B| \), \( |A \cap B| \), \( |A \setminus B| \), \( |B \setminus A| \).

4. (i) Write the following using set notation:
   The set \( G \) is the set of all odd integers which are greater than 22 and are not divisible by 5.
   (ii) With \( G \) as described in part (i), classify each of the following statements as true or false, giving reasons.
   (a) \( G \subseteq \mathbb{Z} \),
   (b) \( \mathbb{Z} \subseteq G \),
   (c) \( G \cap \mathbb{Z} \neq \emptyset \),
   (d) \( \mathbb{Z} \setminus G \) is the set of all even integers less than 22 which are divisible by 5.

5. Let \( A = \{a, b, c\} \), \( B = \{a, \{a\}, \{b, c\}\} \) and \( C = \{\{a, b\}, b, c\} \).
   (i) What are \( |A| \), \( |B| \) and \( |C| \)?
   (ii) Write down \( A \cup B \), \( A \cup B \cup C \), \( A \cap B \), \( A \cap C \) and \( B \cap C \).
   (iii) Write down \( A \setminus B \), \( B \setminus A \), \( A \setminus C \), \( B \setminus C \) and \( C \setminus A \).
   (iv) Which of the following statements are true? Give reasons!
   (a) \( A \subseteq B \),
   (b) \( B = C \),
   (c) \( \{a\} \in A \),
   (d) \( \{a\} \in B \),
   (e) \( \{\{a\}\} \subseteq B \),
   (f) \( A \subseteq C \),
   (g) \( \{a, b\} \subseteq A \),
   (h) \( \{a, b\} \subseteq C \),
   (i) \( \{a, b\} \in C \),
   (j) \( \{\{a, b\}\} \subseteq C \).

6. Let \( A \), \( B \) and \( C \) be any three sets. Prove that
   \((A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)\).

7. Prove that if \( A \) and \( B \) are subsets of \( X \), then
   \( X \setminus (A \cup B) = (X \setminus A) \cap (X \setminus B) \).