1. Define \( f : \mathbb{N} \rightarrow \mathbb{N} \) by \( f(x) = x + 1 \). Determine whether or not \( f \) is
   (a) one-to-one; (b) onto.

2. Each of the following sets of pairs may or may not represent a function from \( \{1, 2, 3\} \) to \( \{a, b, c, d\} \):
   \[
   \{(1, d), (2, b), (3, d)\}, \quad \{(1, c), (2, a), (3, b)\}, \quad \{(1, a), (3, b)\}
   \]
   \[
   \{(1, a), (1, c), (3, d)\}, \quad \{(2, b), (3, c), (1, d)\}
   \]
   (i) Identify the sets which represent functions and determine which of these are one-to-one.
   (ii) Explain clearly why each of the sets does or does not represent a function.
   (iii) Explain clearly why each of the sets does or does not represent a one-to-one function.

3. (i) Let \( A = \{-1, 2, 3, 5, 7, 11\} \) and let \( B = \{1, 2, \ldots, 200\} \). Is the function \( f : A \rightarrow B \) given by \( f(x) = x^2 \) one-to-one?
   (ii) Now suppose that \( A = \{-2, -1, 2, 3, 5, 7, 11\} \) and \( B = \{1, 2, \ldots, 200\} \). Is the function \( f : A \rightarrow B \) given by \( f(x) = x^2 \) one-to-one?

4. Use arrow diagrams to write down all the functions from the set \( \{1, 2\} \) to the set \( \{a, b, c\} \). How many are there? How many one-to-one functions and how many onto functions?

5. Let \( A = \{1, 2, 3\} \) and \( B = \{a, b, c\} \). Write down all the one-to-one correspondences between \( A \) and \( B \).

6. For each of the following functions, determine whether it is one-to-one and/or onto.
   (i) \( f : \mathbb{Z} \rightarrow \mathbb{Z} \) given by \( f(x) = x^2 \).
   (ii) \( f : \mathbb{N} \rightarrow \mathbb{Z} \) given by \( f(x) = x^2 \).
   (iii) \( f : \mathbb{N} \rightarrow \mathbb{N} \) given by \( f(x) = x^2 \).
   (iv) \( f : \mathbb{R} \rightarrow \mathbb{R} \) given by \( f(x) = x^2 \).
   (v) \( f : \mathbb{R}^+ \rightarrow \mathbb{R} \) given by \( f(x) = x^2 \).
   (vi) \( f : \mathbb{R}^+ \rightarrow \mathbb{R}^+ \) given by \( f(x) = x^2 \).
   In this question \( \mathbb{R}^+ = \{x \in \mathbb{R} \mid x > 0\} \).

7. Let \( C_{n,k} \) be the set of subsets of \( \{1, \ldots, n\} \) of size \( k \).
   (i) Describe explicitly \( C_{5,2} \). What is the cardinality of \( C_{5,2} \)?
   (ii) Describe explicitly \( C_{4,2} \cup C_{4,1} \). What is the cardinality of \( C_{4,2} \cup C_{4,1} \)?
   (iii) Define \( f : C_{5,2} \rightarrow C_{4,2} \cup C_{4,1} \) by \( f(A) = A \setminus \{5\} \). Describe \( f \) explicitly on the elements of \( C_{5,2} \).
Problem Set 3

1. Let $A = \{a, b, c\}$ and $B = \{1, 2, 3, 4\}$. Which of the following arrow diagrams determine functions from $A$ to $B$? Which of them determine a one-to-one functions?

   (i) ![Diagram 1]
   (ii) ![Diagram 2]
   (iii) ![Diagram 3]
   (iv) ![Diagram 4]

2. (i) Use arrow diagrams to write down all the functions from the set $\{1, 2\}$ to $\{a, b\}$.
   (ii) Let $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c\}$. Let $f : A \rightarrow B$ be the function given by the 4-tuple $(b, a, c, a)$. Draw the arrow diagram of $f$. Is $f$ injective? Is $f$ surjective?
   (iii) Let $A = \{a, b, c\}$ and $B = \{1, 2, 3\}$. Does the following set of pairs
   \[ F = \{(a, 2), (b, 1), (c, 2)\} \]
   represent a function $g$ from $A$ to $B$.
   If so, is $g$ injective? Is $g$ surjective?

3. Let $A = \{0, 1, 2, 3\}$ and $B = \{0, 1, 2, 3, 4, 5\}$, and consider the function $f : A \rightarrow B$ with rule $f(x) = x^2 - 4x + 4$ for all $x \in A$.
   (i) Draw an arrow diagram to represent $f$. Write $f$ as a set of ordered pairs of integers.
   (ii) Is $f$ injective? Give reasons.
   (iii) Find the image of $f$. Is $f$ surjective? Give reasons.
   (iv) Find a set $C$ such that with the same domain $A$,
   \[ f : A \rightarrow C, \ f(x) = x^2 - 4x + 4 \]
   is surjective.

4. Use arrow diagrams to write down all the functions from the set $\{1, 2, 3, 4\}$ to the set $\{a, b\}$. How many are there? How many one-to-one functions and how many onto functions?