1. If possible, compute $|A \cup B \cup C|$ from the given information. If it is not possible, explain why.

   (i) $|A| = 12$, $|B| = 13$, $|C| = 7$, $|A \cap B| = 5$, $|A \cap C| = 4$, $|B \cap C| = 3$ and $|A \cap B \cap C| = 2$.

   (ii) $|A| = 12$, $|B| = 13$, $|C| = 7$, $|A \cap B| = 10$, $|A \cap C| = 7$, $|B \cap C| = 13$ and $|A \cap B \cap C| = 2$.

   (iii) $|A| = 12$, $|B| = 13$, $|C| = 7$, $|A \cap B| = 10$, $|A \cap C| = 4$, $|B \cap C| = 6$ and $|A \cap B \cap C| = 2$.

2. You have 6 different apples and 3 different boxes. In how many ways can you arrange the apples in the boxes so that each box contains at least one apple? Hint: Let $X$ be the set of all possible arrangements of the apples in the boxes and let $A_i$ be the set of arrangements for which the $i$-th box is empty. Use one of the forms of the Inclusion-Exclusion Principle.

3. In a group of 400 students, 300 are doing mathematics, 250 are doing physics and 200 are doing chemistry. Furthermore, 210 are doing mathematics and physics, 120 are doing mathematics and chemistry and 80 are doing physics and chemistry. Only 40 are doing all three subjects. How many students are not doing any of these subjects?

4. In a group of 50 participants at a recent international meeting, 30 speak English, 18 speak German, 26 speak French, 9 speak both English and German, 16 speak both English and French, 8 speak both French and German, and 47 speak at least one of English, French or German.

   (i) How many people in the group cannot speak English, French or German?

   (ii) How many people in the group can speak all three languages?

5. How many numbers between 1 and 100 (inclusive) are divisible by at least one of the numbers 3, 5 or 7?

6. Given $A = \{1, 2, 3, 4\}$. How many permutations $f : A \rightarrow A$ have the property that $f(i) = i$ for at least one value of $i$?
1. (i) In how many ways can three married couples be seated in a row of six chairs?
(ii) In how many ways can three married couples be seated in a row of six chairs with at least one couple seated together?
(iii) In how many ways can three married couples be seated in a row of six chairs with no couple seated together?

2. Consider the letters in the word KANGAROON.
   (i) How many ways to rearrange the letters?
   (ii) How many ways to rearrange the letters so that the 2 A’s are together?
   (iii) How many ways to rearrange the letters so that at least one of the pairs AA, NN and OO is together?
   (iv) How many ways to rearrange the letters so that none of the pairs AA, NN and OO is together?

3. (i) Among a group of 1500 students, 350 study Discrete Mathematics, 620 study Statistics and 870 study Calculus. If 120 study both Discrete Mathematics and Statistics, 200 study Discrete Mathematics and Calculus, 350 study Statistics and Calculus, and 50 study Discrete Mathematics, Statistics and Calculus, how many of these students are not studying Discrete Mathematics, Statistics or Calculus?
   (ii) While taking a 6-week summer mathematics class, Alison frequently had dinner with seven friends from her hometown. She ate dinner with each friend (exactly) 15 times, every pair of friends 8 times, every set of three friends 6 times, every set of four 5 times, every set of five 4 times, and every set of six 3 times, but she never ate with all seven at once. On how many days did Alison have dinner with none of these friends? (Count a week as 7 days.)

4. (i) In a class of 800 students, 450 are doing Integral Calculus, 250 are doing Discrete Mathematics and 143 are doing both subjects. How many are not doing either subject?
   (ii) How many numbers between 1 and 250 are divisible by at least one of 3, 7 or 11?