

1. Consider the function given by

$$f(x) = 2x^3 + 3x^2 - 72x \text{ for } -5 \leq x \leq 7.$$

List the co-ordinates (x, y) of the locations of the following critical points:

Co-ordinates of point	Classification of point
$(-5, 185)$	local minimum
$(7, 329)$	global maximum
$(-4, 208)$	local maximum
$(3, -135)$	global minimum
$(-\frac{1}{2}, \frac{73}{2})$	inflexion

2. Consider the function of two variables given by

$$f(x, y) = 16x - 3x^2 + 4y + 2xy - 5y^2 - 7.$$

Write down the following:

$f_x(x, y) = -6x + 2y + 16$	$f_y(x, y) = 2x - 10y + 4$
$f_{xx}(x, y) = -6$	$f_{yy}(x, y) = -10$
$f_{xy}(x, y) = 2$	$\Delta f(x, y) = 56$
Co-ordinates (x, y, z) of critical point(s):	$(3, 1, 19)$
Classification(s) of the above points(s):	local maximum

3. A biologist believes that her data in the table below satisfies the equation $y = a + bx$, where a and b are undetermined constants to be found.

$n = 8$	x	y	x^2	xy
	1	5	1	5
	2	7	4	14
	3	11	9	33
	4	12	16	48
	5	15	25	75
	6	18	36	108
	7	19	49	133
	8	21	64	168
Σ	36	108	204	584

In fact, there are no values of a and b which for which her data fits the formula exactly. However, she is content to settle for a “least squares line of best fit.” By filling out the table above and summing, write down two simultaneous linear equations for a and b . Solve the equations and use the resulting formula to estimate y when $x = 9$.

<p><i>Simultaneous equations:</i></p> $8a + 36b = 108, 36a + 204b = 584$	<p><i>Line of best fit:</i></p> $y = 3 + \frac{7}{3}x$
<p>When $x = 9$, $y \approx 24$</p>	

4. Evaluate (and simplify!) the following series:

(a) $5 + 10 + 15 + \dots + 250$

6375

(b) $\sum_{i=1}^{50} (1.23)^{i-1}$

135992

(c) $\lim_{n \rightarrow \infty} \sum_{j=1}^n (0.55)^{j-1}$

$\frac{20}{9} = 2.\dot{2}$

(d) $\sum_{k=1}^n (4k^3 + 6k^2)$

$n(n+1)(n^2+3n+1)$

(e) $\sum_{k=6}^{75} \left[\cos\left(\frac{\pi}{2}(k+1)\right) - \cos\left(\frac{\pi}{2}k\right) \right]$

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