

Welcome!

You should ask yourself “why am I doing this course”? Unfortunately the answer for most students is “because I have to”. But allow me to convince you otherwise... “Mathematics is the science of logic”, so if you want to make sense, you need to think mathematically.

In this course, we explore an array of applications of calculus to the real world. Indeed mathematics is everywhere, from physics to dentistry, from astronomy to psychology and finance. And there will be maths involved in your field of research one day, and this course is designed as an introduction to the various tools that you may well need one day!

Below is some important information about the course, please take the time to read through it. At the end is a (draft) syllabus for you to follow your progress and test your understanding. Most of all, I hope you can enjoy the beauty of calculus and it’s usefulness in our world, and that you exceed your expectations in your own ability of mathematics in this unit of study.

Web Site

It is important that you check the website for this unit of study regularly. It may be found by following the links from the University of Sydney front page, or by going directly to:

<http://www.maths.usyd.edu.au/u/UG/SS/SS1011/>

Lectures

Lectures begin on Monday 9th January 2012 (please take note of the university summer school convention: this week is deemed as “**week 2**” even though it is our first week of classes). There will be two 2 hour lectures per week (George Papadopoulos will be lecturing), occurring on Monday and Tuesday at 10am, in Carslaw 375. Lecture slides for the course are available on the course website.

Textbook

The reference text for this course is *Calculus*, James Stewart, 6th [Metric International] Edition, Thompson - Brooks/Cole (2008), ISBN-13: 978-0-495-01160-6. There are also some course notes relevant to the course which you may find useful: *Applications of Calculus* (Lecture notes for MATH1011), School of Mathematics and Statistics, University of Sydney. Available from Kopystop, 55 Mountain Street Broadway.

Calculators

You may use programmable and/or graphics calculators to aid you throughout the course, however these are not permitted for exam use¹; we recommend that you do not *rely* upon these too much! The recommended calculator for this course is the *Casio fx-82 AU*.

Another resource you may find incredibly useful is *Wolfram Alpha*, which is a free online computational engine which can perform many mathematical calculations (including ones involving functions and calculus that you will be experiencing in this course). It can be found at <http://www.wolframalpha.com>. Whilst you are encouraged to use this service to check your answers, remember that you must also know how to show your working.

¹For a list of university exam approved calculators, refer to http://sydney.edu.au/current_students/student_administration/examinations/students.shtml

Tutorials

Tutorials begin in week 2, and occur directly after the lectures at noon. Tutorial sheets are available from the course website. Please bring your own copy to each tutorial! Solutions to each week's exercise sheets will be posted online later in the week after the Tuesday tutorial.

Stream	Location	Tutor
1	E. Av. 119	George Papadopoulos
2	E. Av. 120	Collin Zheng
3	E. Av. 121	Audrey Deheinzelin

Tutorial Allocation

George Papadopoulos (Stream 1, E. Av. 119)	Collin Zheng (Stream 2, E. Av. 120)	Audrey Deheinzelin (Stream 3, E. Av. 121)
ABE, HITOMI	ALSULAIMAN, ALI OJAYM A	BAE, KYOUNG WON
BARNAWI, HISHAM IBRAHIM M	BE, KARREN	CALDERON, ALEX ANTHONY
CHAHAL, HARDIP	CHENG, CHING YAN SERENE	CHIANDOTTO, LAURA
CHUA, MELISSA WAN YING	COONEY, ELIZABETH ANNE	COOPER-FYFE, ELIZABETH
DEASEY, HUGH DAVID WILLIAM	FARRELL, DAVID JAMES	GAMIELDIEN, MOHAMED
GEORGULOPOULOS, HARRY	GODWIN, ALEXANDRA	GRUNDY, HANNAH CATHERINE
HIOE, SIDNEY NICHOLAS	HORSFALL, DANIEL CRAIG	HYUN, JUNG HEUN
KORECKI, TATJANA	LEE, JESSICA	LOIKO, JULIA
MANG, CHUN SING JASON	MCCABE, MATTHEW	METELERKAMP, KIRSTY
MO, MAX	MORGAN, COURTNEY ADELE	NASIM-JAN, NISHEY
PATU, PANIANI NIKOLAS	PEDLOW, STEPHANIE	RAHMAN, AHSAN
ROBBINS, ROBERT	ROBERTS, SAMANTHA	SATO, KENTARO
SCHLEGEL, MATTHEW	SNOWDEN, LUCY MAY	SOMA DEVAN, KUHEN NAGAPPAN
SOMASUNDARAM, AGALYA	STOREY-LEWIS, BROOKE SHEREE	TAKATA, AKI
TRAN, LOLA	TSUNAKAWA, YUKI	WU, LI
WULF, INGRID NICOLE	XIANG, LI JUN	ZHU, WINA

Contact

Please do not hesitate to contact George or the tutors for any additional assistance. George will be having a consultation hour at 1pm each Tuesday after the tutorial in his office, which is located in Carslaw (level 4) 495. Alternatively, you can email George or the tutors:

georgep@maths.usyd.edu.au
collinz@maths.usyd.edu.au
adeh1320@uni.sydney.edu.au

Assessment

Your final raw mark for this unit will be calculated as follows:

5%	Participation/Attendance
20%	Quizzes (two, 10% each)
25%	Assignment
50%	Final Exam

The participation mark is given to students who (a) attend the lectures, (b) attend the tutorials, and (c) submit written solutions to the preparatory questions before the tutorials. Sample quizzes will be released online in weeks 3 and 5, and the quizzes will take place in the first half hour of your Monday tutorials in weeks 4 and 6. The assignment will be released online in week 2, and is due at the start of the final lecture in week 7 on 14/02/2011. We **STRONGLY** recommend that you start working on the assignment **IMMEDIATELY!** It is a long assignment that cannot be done in one sitting, and should be done as the course progresses week by week. Past MATH1011 exam papers will also be available on the University library website. The exam will be an hour and a half long, and will be held during week 8.

Guidelines for a decent pass mark

You will be expected to be fluent in any topics that have already been covered in high school. These include the majority of Periodic Functions, Single Variable Differential Calculus, Series and Single Variable Integral Calculus. In the cases where these topics have extensions or advanced methods that are normally taught in extension classes (eg. Extension I or II in NSW) in high school, you should be able to perform simple examples fairly well. Topics that are not taught in school (but have little extension from high school) such as Data Scaling and Least Squares should be picked up fairly easily, as these topics do not have advanced university concepts, but are rather mostly extensions of high school maths (where occasionally some university maths is used for derivation or justification). Topics that introduce strictly tertiary level material such as Multi Variable Differential and Integral Calculus are obviously taught and learned at a much slower pace.

We expect that students who pass should become familiar with the basic concepts, definitions, formulas and methods of these topics, however there are some more advanced concepts and techniques that are aimed towards credit/distinction students. In the table below, specific outcomes have been divided into [Pass] and [Credit/Distinction] categories, so that you know what is the bare minimum knowledge to pass this course, and what are the more challenging concepts that will lead to a credit/distinction grade. In any case, the lecturer will make specific mention during lectures what topics are considered basic pass knowledge and what are considered credit/distinction knowledge. If in doubt (as always), just ask!

The **assumed knowledge** for the course is all maths up to year 12 NSW HSC two unit (Advanced) mathematics (or equivalent). The below table lists outcomes, most (but not all) of which are specific to this course and are outside the assumed knowledge (some overlap with the high school syllabus). If you are having trouble with the course due to lack of background maths, then please seek the help of George or the tutors during tutorials or consultation hours.

(Draft) SS1011 Syllabus Outcomes

Section <i>Name and outline</i>	Outcomes <i>Students aiming for a decent [Grade] mark should be able to:</i>
<p>1. Periodic Functions</p> <p><i>Students start learning about periodic functions in general, including their mathematical and graphical representations. This notion is then extended to trigonometric functions which are taught in high school, and one looks at the various applications to nature.</i></p>	<p>[Pass]</p> <ul style="list-style-type: none"> ● Graph periodic functions given by a periodic rule ● Write down the rule given the graph of a periodic function ● Use periodicity to evaluate functions evaluated at arbitrarily large values ● Stretch and shift sinusoidal functions ● Deduce the amplitude, period, “mean value” and “starting point” of a sinusoid ● Write down the equation of a graphed sinusoid ● Combine sinusoidal functions’ equations and sketch their graphs <p>[Credit/Distinction]</p> <ul style="list-style-type: none"> ● Define what it means for a function to be periodic ● Understand that sinusoidal functions are periodic functions made by adding trigonometric functions of the same period ● Sketch sinusoidal functions with any given amplitude, period, “mean value” and “starting point”
<p>2. Data Scaling</p> <p><i>Linear data is easiest to interpret and represent, but data in nature is often non-linear. In this topic we explore how to transform nonlinear data into linear data using logarithmic transformations.</i></p>	<p>[Pass]</p> <ul style="list-style-type: none"> ● Plot discrete empirical data and recognise a linear relationship by drawing a line of best fit ● Empirically write down the equation of the line of best fit of linear data ● Discover nonlinear data, and use logarithmic (or other) transformations to linearise it ● Derive allometric or exponential laws for nonlinear data <p>[Credit/Distinction]</p> <ul style="list-style-type: none"> ● Understand the meaning of proportionality and the application to linear functions ● Apply logarithmic scaling to real-world examples

<p>3. Single Variable Differential Calculus</p> <p><i>Many concepts from differential calculus in high school will be re-visited in this topic. Particular emphasis is placed on geometrical interpretations of derivatives as well as using differential calculus to solve real world optimisation problems.</i></p>	<p>[Pass]</p> <ul style="list-style-type: none"> • Calculate derivatives of any order, using the chain, product and quotient rules on a variety of elementary functions • Find the equation of a tangent line to a curve at a point • Reconcile the first/second derivatives of displacement-time functions as instantaneous velocity/acceleration respectively • Correlate values of the first derivative with a function that is increasing/decreasing/stationary (at a point) • Check singularities and endpoints and use differential calculus to find critical points/values of functions • Differentiate between global and local extrema • Use the second derivative to determine concavity and find candidate points of inflexion • Apply gathering data on critical points to aid in curve sketching • Apply the above skills to solve optimisation problems in one variable, verifying the validity of physical results <p>[Credit/Distinction]</p> <ul style="list-style-type: none"> • Understand the geometry of the derivative as the instantaneous rate of change and slope of the tangent to a point on a curve • Explain how the notion of the derivative (or slope of the tangent to a point on the curve) is derived from the gradient of the secant connecting two points, and that the derivative is the limit as the distance between the points becomes arbitrarily small • Define the notion of a critical point/value • Check candidate inflexion points using either the first or second derivative test or table method
--	---

<p>4. Introduction to Multi Variable Differential Calculus</p> <p><i>We introduce multi variable differential calculus with functions of two variables (surfaces) in three dimensions. As students learn about partial derivatives, notions from single variable differential calculus are analogised and extended. Emphasis is placed on geometrical interpretation and problem solving applications.</i></p>	<p>[Pass]</p> <ul style="list-style-type: none"> • Recognise functions of 2 (or more) variables and evaluate them at points • Draw contours or cross sections of functions of 2 variables • Analogise the tangent line in two dimensions with the tangent plane in three dimensions • Calculate first and second order partial derivatives and evaluate them at points • Calculate the equation of the tangent plane for functions of two variables at a point • Analogise the notion of critical points to functions of several variables: turning, saddle, singular, and boundary points • Check singularities, boundaries and use partial derivatives to find critical points/values of surfaces • Use the discriminant to classify calculus-type critical points • Gather data on critical points to find global extrema of functions of several variables • Apply the above skills to solve optimisation problems in several variables, verifying the validity of physical results <p>[Credit/Distinction]</p> <ul style="list-style-type: none"> • In 3 dimensions, understand that a point (x, y) in the domain is input into f to obtain $z = f(x, y)$ on a surface in xyz-space • Know the equations and be able to graph basic surfaces: planes, (semi) spheres, paraboloids, ellipsoids, hyperboloids, and surfaces of revolution • Deduce geometrical properties of surfaces from contour plots or cross sections • Combine analogous geometrical properties of simpler functions of one variable to deduce properties of surfaces composed of these • Compare and contrast geometric/algebraic properties of the second derivatives and their tests for functions of one and two variables
---	--

<p>5. The Method of Least Squares</p> <p><i>We explore an important application of calculus to statistics. How can we be sure that the line of best fit is indeed the “best fit”? Optimisation methods are used to derive the line of best fit, for both linear and non-linear data.</i></p>	<p>[Pass]</p> <ul style="list-style-type: none"> • Given tabulated data, plot data points and draw a line of best fit by eye accurately • Tabulate the required sums involving the x and y data points • Memorise and solve the simultaneous equations that yield the y-intercept and gradient of the line of best fit using the tabulated sums <p>[Credit/Distinction]</p> <ul style="list-style-type: none"> • Understand that a “line of best fit” is obtained when the “error function” of two variables (the y-intercept and the gradient) of the line with respect to the data is minimised using multi variable optimisation methods • Apply the method of least squares to data scaling problems to analyse the “curve of best fit” of nonlinear data
<p>6. Series</p> <p><i>Another topic mostly revised from high school with several extensions and new tricks; applications to the real world are explored. This topic is a crucial preface to Integral Calculus.</i></p>	<p>[Pass]</p> <ul style="list-style-type: none"> • Convert series into sigma notation and vice versa • Evaluate series involving the sum of n integers, perfect squares and perfect cubes • Define arithmetic/geometric progressions as special types of series: adjacent terms have a common difference/ratio • Recognise and evaluate arithmetic and geometric series • Evaluate infinite geometric series • Apply series in solving real world problems <p>[Credit/Distinction]</p> <ul style="list-style-type: none"> • Where applicable, recognise the use of collapsing sums to simplify and evaluate series • Understand when infinite geometric series converge • Explore the notion of other (non geometric progression) infinite series that still converge, and can be used to define fundamental constants

<p>7. Single Variable Integral Calculus</p> <p><i>Students revisit their knowledge of integration from highschool in this topic, with advanced extensions as well. Real world applications, as well as new techniques for integrating, are introduced. Most importantly we focus on geometrical interpretation and the relationship with differential calculus.</i></p>	<p>[Pass]</p> <ul style="list-style-type: none"> ● Understand the geometrical interpretation of the integral as the “area under the curve” above a given integration interval ● Reconcile the integral over a time interval of a velocity-time functions as the distance traveled over that time interval ● Realise that rules/formulas for integrating can be derived by applying integration to the rules/formulas for differentiating ● Evaluate definite/indefinite integrals of elementary functions ● Apply integration to solve real world problems ● Reconcile the integral as giving an “algebraic area”, and know how to obtain the “absolute area” ● Evaluate definite and indefinite integrals of compound functions using either integration by substitution or recognition ● Apply integration to calculate the average value of a function or the area between curves ● Differentiate between proper and improper integrals <p>[Credit/Distinction]</p> <ul style="list-style-type: none"> ● Explain how the notion of the integral (or area under the curve) is derived from a finite series of the areas of discrete approximating rectangles, and that the integral is the limit as these rectangles become arbitrarily numerous and small ● Use Riemann sums in simple cases to approximate integrals, areas or displacements over a time interval ● Perceive integration as the reverse of differentiation in the light of the Fundamental Theorem of Calculus ● Use a constant of integration for indefinite integrals and understand why it is important ● Apply integration by parts to solve relevant integrals involving products in the integrand ● Understand that the chain rule for integration is derived from the chain rule for differentiation ● Understand that improper integrals are “made proper” through evaluating limits of proper integrals ● Evaluate improper integrals as limits of proper integrals over infinite domains or domains that contain a singularity of the integrand, partitioning the domain if necessary ● Define the notion of an improper integral converging/existing or diverging/not existing
--	---

<p>8. Introduction to Multi Variable Integral Calculus</p> <p><i>We introduce multi variable integral calculus by firstly introducing double integrals as a way of integrating functions of two variables (surfaces) in three dimensions. As students learn about iterated integrals, many notions from single variable integral calculus are analogised and extended. Again, emphasis is placed on geometrical interpretation as well as problem solving applications.</i></p>	<p>[Pass]</p> <ul style="list-style-type: none"> • Understand the geometrical interpretation of the double integral as the “volume under the surface” above a domain in the xy-plane • Conceptualise the multiple integral as a series of iterated single integrals using the “working from the inside out” approach • Interpret integrals geometrically, and if possible, evaluate without integrating using elementary geometry <p>[Credit/Distinction]</p> <ul style="list-style-type: none"> • In the case of two variables, describe a double (volume) integral as adding a series of thin slices • Define and apply Fubini’s Theorem (for rectangular domains) to simplify multiple integrals • Analogise (from the single integral) the notion of “algebraic” and “geometric” volumes • Where applicable, apply separation of variables to separate multiple integrals into multiples of single integrals • Evaluate multiple integrals (over rectangular and non-rectangular domains) using techniques of single variable integration • Write concise mathematical descriptions of simple regions using set notation and/or inequalities and draw these • Decide which direction to integrate first if the integration region is bounded by two or more curves regions • Apply multi variable integration to solve physical problems (such as finding volumes)
--	--