
Practice Questions for Quiz 1

Basic Knowledge

About half of the questions on the quiz will be of this type, and if you have attended all the lectures, you should have no trouble with this type of question.

Give the general solution to these equations.

1. $\frac{dP}{dt} = 5P.$

2. $3\frac{dP}{dt} - 2P = 0.$

3. $3Q'(t) + 4 = 0.$

4. $3X' + 4X = 0.$

5. $X_{n+1} = 5X_n.$

6. $P_{n+1} - P_n + 5 = 0.$

7. $3Q_{n+1} + 2Q_n = 0.$

Give the particular solution to these equations that satisfies the given condition.

8. $3\frac{dP}{dt} - 2P = 0$ with $P(0) = 2.$

9. $3Q'(t) + 4 = 0$ with $Q(0) = 7.$

10. $3X' + 4X = 0$ with $X(0) = 1.$

11. $7P_{n+1} = 5P_n$ with $P_0 = 3.$

12. $3Q_{n+1} + 2Q_n = 0$ with $Q_0 = 5.$

Find all the equilibrium solutions to these equations.

13. $3\frac{dP}{dt} + 2P + 5 = 0.$

14. $\frac{dY}{dt} = Y^2 + 3Y + 2.$

15. $7Q' + 4Q^2 + 3Q = 1.$

16. $X' = kX + b.$

17. $P_{n+1} = 5P_n - 3.$

18. $3P_{n+1} + 2P_n + 5 = 0.$

19. $7Q_{n+1} + Q_n^2 - 4Q_n = 10.$

20. $2Y_{n+2} + 3Y_{n+1}^2 + 2Y_n + 1 = 0.$

Numerical Calculations

There will be a few questions that require a numerical calculation.

Exponential functions and geometric progressions.

1. A substance decays according to the differential equation

$$\frac{dC}{dt} = -kC$$

where $k = 0.02$. What is the half-life?

2. Sales of a popular product are growing according to

$$\frac{dS}{dt} = aS.$$

If $S(0) = 100$ and $S(20) = 350$ calculate the value of a .

3. Attendance at lectures decays according to

$$L_{n+1} = rL_n$$

where $L_1 = 350$ and $L_{13} = 100$. What is the value of r ?

4. The incidence of a new disease satisfies

$$D_{n+1} = rD_n.$$

If $r = 1.02$ and $D_0 = 50$ when does D_n exceed 10 times its initial value?

Calculating sequences term by term.

5. Starting with $X_0 = 1$ calculate the next 3 terms in $X_{n+1} = 2X_n^2$.
6. Starting with $X_0 = 20$ calculate the next 3 terms in $5X_{n+1} = X_n - 20$.
7. The popularity of a club satisfies

$$P_{n+1} = rP_n.$$

If $r = 1.2$ and $P_0 = 5$ when does P_n exceed 10 times its initial value?

Stability

There will be a few questions that check your knowledge of stability and there will usually be more than one way of working out the answer.

1. Determine the stability of the equilibria of $P'(t) = P^2 - 5P + 6$.
2. Determine the stability of the equilibria of $X_{n+1} - X_n = e^{-X_n} - 0.3$.
3. Determine the stability of the equilibria of $5Q - Q' = Q^2 + 4$.
4. Determine the stability of the equilibria of $4Y_{n+1} + Y_n = 4 + Y_n^2$.

Logical thinking: Multiple Choice Questions

There will only be a couple of multiple choice questions, and they will probably be the hardest questions in the quiz. They will require you to combine ideas from different weeks to determine the correct option.

For each multiple choice questions, only one of the four statements is correct and can be logical deduced from the information given.

1. Consider the difference equation $X_{n+1} = X_n^3$. Which one of the following statements is correct?
 - (a) There are 3 stable equilibria.
 - (b) There are 2 stable and 1 unstable equilibria
 - (c) There are 1 stable and 2 unstable equilibria.
 - (d) There are 3 unstable equilibria.
2. The differential equation $P'(t) = 2P^2 + bP + 3$ is known to have two different equilibrium solutions. Which one of the following statements must be true?
 - (a) $|4P_{eq} + b| < 1$
 - (b) $4P_{eq} + b < 0$
 - (c) $b^2 > 24$
 - (d) $b^2 > P_{eq}$
3. Consider the recurrence relation $X_{n+1} = rX_n^2$ where $r > 0$. Which one of the following statements is true?
 - (a) The total number of equilibrium solutions depends on r .
 - (b) The equilibrium at $X_{eq} = 0$ is stable if and only if $|r| < 1$.
 - (c) Exactly one of the equilibrium solutions is unstable.
 - (d) All equilibria are stable.

Advice

- There are several blank pages provided for your rough work. The rough work will NOT be marked. There is plenty of room to draw any sketches you might need or to rearrange the equations.
- You have 40 minutes to do 15 questions, so that is on average between 2 and 3 minutes per question.
- Some questions are much easier than other questions. The questions are NOT necessarily in order of difficulty! The questions are NOT necessarily in the same order that the material was covered in the course!
- If you are spending a lot of time on one question and getting nowhere, consider moving on to another question.
- There may be more than one way to do some questions, think about which method you are most confident with, or which method you think will be fastest, or which method is least likely to cause you to make an error!