
ASSIGNMENT 2 **Due: 2pm, Monday, 6 February 2012**

This assignment is worth 5% of the assessment for this unit of study.
Your assignment must be stapled inside a manilla folder with your name on it.

1. Let the function f be defined by $f(x) = x^2 - 12x + 5$.
 - (a) [1 Mark] By evaluating $f(0)$ and $f(0.5)$, explain why $f(x) = 0$ has a solution between 0 and 0.5.
 - (b) [2 Marks] Find the equilibrium condition for $X_{n+1} = G(X_n) = \frac{1}{12}(5 + X_n^2)$ and show this can be rewritten as $f(X_{eq}) = 0$.
 - (c) [3 Marks] Find $G'(X_n)$ and show that the fixed point of this iteration scheme between 0 and 0.5 is stable.
Hint: plot $G'(X_n)$
 - (d) [1 Mark] Starting with $X_0 = 0$, find the solution of $f(x) = 0$, correct to four decimal places.

2. A restricted access lake is stocked at the beginning of this year with 500 fish. The estimated maximum size of the fish population for the lake is 8000. Let $P(t)$ be the number of fish in the lake at time t years from the beginning of this year. Suppose that the size of the fish population is modelled by the differential equation:

$$8000 \frac{dP}{dt} = P(8000 - P).$$

- (a) [4 Marks] Using the method of partial fractions, solve the differential equation and show that the general solution has the form

$$\ln |P| - \ln |8000 - P| = t + C,$$

where C is an arbitrary constant.

- (b) [1 Mark] Determine the value of C using the initial condition.
- (c) [2 Marks] How long does it take for the population to reach half of the maximum capacity of the lake? Estimate the size of the fish population after three years.
- (d) [1 Mark] Comment on the long term behaviour of the population.