Practice Questions for Quiz 2

First Order Linear Equations

Give the general solution to these equations.

1. $P'(t) = 5P - 3$.
2. $3P' + 2P + 5 = 0$.
3. $P_{n+1} = 5P_n - 3$.
4. $3P_{n+1} + 2P_n + 5 = 0$.

Give the particular solution to these equations that satisfies the given condition.

5. $P'(t) = 7P - 4$ with $P(0) = 1$.
6. $2P' + 3P + 4 = 0$ with $P(0) = 0$.
7. $P_{n+1} = 3P_n - 5$ with $P_0 = 2$.
8. $2P_{n+1} + 5P_n - 14 = 0$ with $P_0 = 9$.

Partial Fractions

Decompose these expressions into partial fractions and then integrate the result.

1. $\dfrac{2}{(x-3)(x-5)}$.
2. $\dfrac{11}{(x-4)(x+7)}$.
3. $\dfrac{1}{x^2+5x+6}$.
4. $\dfrac{5}{2x^2+7x+3}$

Separation of Variables

Find the functions $P(t)$ that solve each of the following problems. You must rearrange your solution to make P the subject.

1. $P'(t) = P^2t^3$ with $P(0) = 1$.
2. $P'(t) = e^{-P}e^t$ with $P(0) = \ln(5)$.
3. $P'(t) = P^2e^t$ with $P(0) = 1$.
4. $P'(t) = e^{-P}t^3$ with $P(0) = 0$.

Find the general solution to each of the following problems. In this case you do NOT need to rearrange your answer.

5. $P'(t) = (P^2 - 4)(t^2 - 4)$.
6. $P'(t) = \dfrac{P^2 - 4}{t^2 - 4}$.

Second Order Linear Equations

Find the general solution to the following second order linear equations

1. $P''(t) - 15P'(t) + 54P(t) = 0$.
2. $Q''(t) - 81Q(t) = 0$.
3. $P_{n+2} - 15P_{n+1} + 54P_n = 0$.
4. $Q_{n+2} - 81Q_n = 0$.
5. $x''(t) + 7x'(t) + 12x(t) = 4$.
6. $y''(t) - 2y'(t) - 15y(t) = 3$.
7. $x_{n+2} + 7x_{n+1} + 12x_n = 4$.
8. $y_{n+2} - 2y_{n+1} - 15y_n = 3$.

Find solutions to the following equations that satisfy the given conditions

1. $x''(t) - 7x'(t) + 12x(t) = 0$ with $x(0) = 3$ and $x'(0) = 10$.
2. $x''(t) - 7x'(t) + 12x(t) = 12$ with $x(0) = 3$ and $x'(0) = 10$.
3. $y_{n+2} + 2y_{n+1} - 15y_n = 0$ with $y_0 = 3$ and $y_1 = 1$.
4. $y_{n+2} + 2y_{n+1} - 15y_n = 12$ with $y_0 = 3$ and $y_1 = 1$.

Iteration

1. Find the solution to $x = \frac{1}{2}\cos(x)$ to three decimal places.
2. Find the solution to $x = \frac{1}{10}e^x$ to three decimal places.
3. The equation $e^x + x = 9$ has a solution close to 2.

   Which of the following iteration schemes is useful in finding this solution?

   $$X_{n+1} = 9 - e^{X_n} \qquad \text{OR} \qquad X_{n+1} = \ln(9 - X_n)$$

   Use the correct scheme to find the solution to three decimal places.

Logistic equation and logistic map

1. If $\ln(p) - \ln(1 - p) = rt + C$ and $p(0) = \frac{1}{3}$ give an exact expression for C.
2. In Robert May's famous 1976 paper he looks at the equation $X_{n+1} = F(X_n)$ with

   $$F(X) = Xe^{r(1-X)}.$$

   For what range of values of r does this iteration have a stable positive fixed point?