

**Week 1-2 Summary for MATH1013 Mathematical Modelling**

<b>Differential Equation</b>	<b>Difference Equation</b>
<b>Model 1:</b> $y'(t) = a$ where $a$ is constant <b>General solution:</b> $y(t) = at + b$ where $b$ is an arbitrary constant	<b>Model 1:</b> $X_{n+1} = X_n + d$ where $d$ is constant <b>General solution:</b> $X_{n+1} = nd + a$ where $a$ is an arbitrary constant
<b>Model 2:</b> $y'(t) = ky$ where $k$ is constant <b>General solution:</b> $y(t) = Ae^{kt}$ where $A$ is an arbitrary constant	<b>Model 2:</b> $X_{n+1} = rX_n$ where $r$ is constant <b>General solution:</b> $X_{n+1} = Ar^n$ where $A$ is an arbitrary constant
<b>Equilibrium condition</b>	
Let $y' = y'' = \dots = 0$ .	Let $X_n = X_{n+1} = \dots = X_{eq}$ .
<b>Stability for first-order DE</b>	
Let $y'(t) = F(y)$ . If $F'(y_{eq}) < 0$ then $y_{eq}$ is stable. If $F'(y_{eq}) > 0$ then $y_{eq}$ is unstable. The <i>sign</i> of the derivative determines stability.	Let $X_{n+1} = F(X_n)$ . If $ F'(X_{eq})  < 1$ then $X_{eq}$ is stable. If $ F'(X_{eq})  > 1$ then $X_{eq}$ is unstable. The <i>magnitude</i> of the derivative determine stability.

Half/Double life =  $\frac{\ln(2)}{|k|}$ ,      where  $k$  is the relative rate of change.