
Tutorial 3

Preparatory Questions

- In each of the following difference equations, write down the next four terms after the given term(s). Try not to use your calculator!
 - $u_{n+1} = 5u_n$ given $u_0 = 2$.
 - $X_{n+1} = 3 + X_n$ with $X_0 = 3$.
 - $P_{n+2} = 3P_{n+1} - 2P_n$ with $P_0 = 1$ and $P_1 = 2$.
- First write down the general solution to each of the following difference equations. (You may need to rearrange the equation first to get it into a form you recognise. Pay careful attention to the subscripts, especially which subscript is larger and which subscript is smaller.) Then calculate the particular solution that satisfies the given condition.
 - $3P_{m+1} = P_m$ with $P_0 = 2$.
 - $3P_m = P_{m-1}$ with $P_0 = 2$.
 - $3Q_{n+1} - 3Q_n + 4 = 0$ with $Q_0 + Q_3 = 10$.
 - $4X_{n-1} + 3X_n = 0$ with $X_1 = 1$.
 - $aY_{n+1} - bY_n = 0$ with $Y_3 = 4$.
 - $A_m - A_{m+1} = d - c$ with $A_1 = c$.
- Use your favourite search engine to investigate the models associated with the names below. For each model try to find out what field it is used in, how many difference equations are used in the model and what the order is of each equation.
 - Nicholson-Bailey
 - Beverton-Holt
 - Ricker
 - Gottman, Swanson & Murray
 - Logistic map
 - Arnold's discrete cat map
- Show that if $X_n = n^2$ that $X_{n+1} - X_{n-1} = 4n$.
 - Substitute $X_n = n^2$ into the expression $X_{n+2} - 2X_{n+1} + X_n$ and simplify as much as possible.
 - Substitute $X_n = n(n+1)$ into the expression $X_{n+2} - 2X_{n+1} + X_n$ and simplify as much as possible.
 - Substitute $X_n = n2^n$ into the expression $X_{n+1} - 4X_n + 4X_{n-1}$ and simplify as much as possible.

Tutorial Questions

5. Wildlife Service records show that the number of female kangaroos in Ecopark is increasing at the rate of 5% each year. At the beginning of 1997 there were 120 females in the park. Let P_n be the number of females in the mob at the beginning of the n th year from 1997. Thus $P_0 = 120$.
- Write down a recurrence relation representing this model.
 - Starting with P_0 and using a calculator, use the recurrence relation to find the anticipated number of females in the mob in each of the next 3 years.
 - Write down the general solution to the difference equation and the particular solution that satisfies $P_0 = 120$.
 - Determine how long it would be before the number of females in the mob doubles.
6. The average weight of baby pigs, recorded in a piggery, increases by 2.3% per week.
- Write down a difference equation for w_n , the average weight after n weeks.
 - The initial average weight is $w_0 = 10.5$ kg. For $n = 1, \dots, 5$ use a calculator to find w_n .
 - After how many weeks will the average weight of the pigs exceed 13.5 kg?
7. Facebook is a social networking website that opened to the public in 2006. comScore is a marketing research company that monitors the popularity of many internet activities by keeping track on a monthly basis of how often the website is used and by how many people. Let F_n represent the number of unique visitors to the Facebook website during month n . According to comScore 19,000 people visited Facebook in December 2006. Let's call that month zero, thus $F_0 = 19,000$. A year later in December 2007 that number had grown to 34,600. Thus $F_{12} = 34,600$.
- Assume the growth in the popularity of Facebook can be modelled as a geometric progression using
$$F_{n+1} = rF_n.$$
Use the given information to calculate the value of r to four decimal places. Then, reinterpret your answer as a percentage growth per month.
 - Which month corresponds to F_{20} ? What value does this model predict for F_{20} (to the nearest hundred)?
 - Which term in the sequence would represent the number of visitors to Facebook in August 2009? What does this model predict for the number of visitors in that month (to the nearest thousand)? *If you have spare time, you can check out how bad this model actually is by looking up current usage trends online.*
8. Use the information from Question 4 to answer the following questions.
- Find a particular solution to $X_{n+1} - X_{n-1} = n$.
 - Find two different particular solutions to $X_{n+2} - 2X_{n+1} + X_n = 1$.
 - Find a particular solution to $X_{n+1} - 4X_n + 4X_{n-1} = 0$. [Advanced: Try to guess some more solutions.]