Tutorial 5/6

Preparatory Questions

1. Give the general solution to each of the following equations.

(a) $P'(t) = 5P - 3$.

(b) $3P' + 2P + 5 = 0$.

(c) $P_{n+1} = 5P_n - 3$.

(d) $3P_{n+1} + 2P_n + 5 = 0$.

2. Give the particular solution to each of the following equations.

(a) $P'(t) = 7P - 4$ with $P(0) = 1$.

(b) $2P' + 3P + 4 = 0$ with $P(0) = 0$.

(c) $P_{n+1} = 3P_n - 5$ with $P_0 = 2$.

(d) $2P_{n+1} + 5P_n - 14 = 0$ with $P_0 = 9$.

3. Find the values of a and b for each of the following partial fraction expansions

(a) $\dfrac{1}{(x-3)(x-5)} = \dfrac{a}{x-3} + \dfrac{b}{x-5}$.

(b) $\dfrac{1}{(x-4)(x+7)} = \dfrac{a}{x-4} + \dfrac{b}{x+7}$.

(c) $\dfrac{1}{x^2+5x+6} = \dfrac{a}{x+2} + \dfrac{b}{x+3}$.

Tutorial Questions

4. You have just bought a bottle of white wine at room temperature ($20°C$). Your thermostatically controlled and extremely expensive refrigerator maintains a constant temperature of $2°C$. You place the bottle in the fridge and 10 minutes later you check and find that the bottle is now $18°C$. Assume Newton's Law of Cooling

$$T(t) = T_{eq} + A\,e^{-kt}.$$

for the temperature (in degrees Celsius) after t minutes.

(a) Calculate T_{eq}, A and k.

(b) How long do you have to wait for the wine to be at $10°C$?

(c) How does this analysis compare with the information at
`http://www.wineinfonet.com/wine-serving-temperature.html`

5. The annual membership of club satisfies $M_{n+1} = 0.9M_n + 10$.

(a) Give the general solution.

(b) Give the particular solution satisfying $M_0 = 5$.

(c) When does M_n first exceed 99?

6. Calculate

$$\int \frac{5}{2x^2 + 7x + 3} dx$$

using partial fractions.

7. An organism has a relative growth rate that is inversely proportional to its age. Assume this can be modelled by the differential equation

$$\frac{1}{L}\frac{dL}{dt} = \frac{k}{t}$$

where L is the length of the organism, and t is the age in years.

(a) Use separation of variables to show that

$$\ln|L| = k \ln|t| + C$$

where C is the arbitrary constant of integration.

(b) Determine the arbitrary constant if $L = 5$ when $t = 1$.

(c) Thus, determine the value of k if $L = 20$ when $t = 2$.

(d) Give the formula for $L(t)$. Assume length and time are both positive.

(e) How long is the organism when $t = 3$?

Partial solutions and/or hints to some of the preparatory questions:

1(b) $P(t) = -\frac{5}{2} + Ae^{-\frac{2}{3}t}$

1(d) $P_n = -1 + A(-\frac{2}{3})^n$

2(b) $P(t) = -\frac{4}{3} + \frac{4}{3}e^{-\frac{3}{2}t}$

2(d) $P_n = 2 + 7(-\frac{5}{2})^n$.

3(c) $a = 1$ and $b = -1$.