Tutorial 7

Preparatory Questions

1. Use separation of variables to find the general solution to each of the following equations. Rearrange your solution to give an explicit formula for $P(t)$.

(a) $P'(t) = P^2t^3$.

(b) $P'(t) = e^{-P}e^t$.

(c) $P'(t) = P^2e^t$.

(d) $P'(t) = e^{-P}t^3$.

2. Use separation of variables and partial fractions to find the general solution to each of the following equations.

(a) $P'(t) = (P^2 - 4)(t^2 - 4)$.

(b) $P'(t) = \dfrac{P^2 - 4}{t^2 - 4}$.

Advanced: Rearrange the solution to give an explicit formula for $P(t)$.

3. Calculate the following integrals:

(a) $\displaystyle\int \frac{1}{x^2 + 5x + 6}dx$.

(b) $\displaystyle\int \frac{1}{5 + 9x - 2x^2}dx$.

(c) $\displaystyle\int \frac{1}{(x-1)(x-2)(x-3)}dx$.

Tutorial Questions

4. Use separation of variables to find the general solution to

$$\frac{dy}{dx} = -\frac{x}{y}.$$

Calculate the value of the arbitrary constant if $y = 2$ when $x = 0$. What geometric object does the particular solution represent?

5. An organism has a relative growth rate that oscillates on a daily basis. Assume this can be modelled by the differential equation

$$\frac{1}{V}\frac{dV}{dt} = k\cos(\frac{2\pi}{24}t)$$

where V is the volume of the organism, and t is the time in hours.

(a) Obtain the general solution using separation of variables.

(b) Determine the arbitrary constant if $V = 1$ when $t = 0$.

(c) Give the formula for $V(t)$.

6. In Tutorial 2 we looked at an organism of length L whose growth rate could be modelled as

$$\frac{dL}{dt} = aL^2 - bL^3$$

where the first term is proportional to surface area, and the second term is proportional to volume. Let $a = 1$ and $b = 1$ in the following calculations.

(a) What is the positive equilibrium L_{eq} for this model?

(b) Find values of p, q and r such that

$$\frac{1}{L^2 - L^3} = \frac{p}{L^2} + \frac{q}{L} + \frac{r}{1-L}.$$

(c) Use separation of variables and the above formula to find the general solution of the differential equation.

(d) Assume that the initial length of the organism (i.e. when it hatches) is 5% of its equilibrium length. Use this information to give an expression for the arbitrary constant?

(e) Give an expression for the time required for the organism to reach 95% of its equilibrium length.

7. Use separation of variables to find the general solution to

$$3P' + 2P + 5 = 0.$$

How does this method of solving the DE compare to the method introduced in Chapter 5?

Partial solutions and/or hints to some of the preparatory questions:

1(b) $P = \ln(e^t + C)$

1(c) $P = -\frac{1}{e^t+C}$.

2(b) $\frac{1}{4}\ln|P-2| - \frac{1}{4}\ln|P+2| = \frac{1}{4}\ln|t-2| - \frac{1}{4}\ln|t+2| + C$

3(b) $\frac{1}{11}\ln|2x+1| - \frac{1}{11}\ln|5-x| + C$