SS2065 Assignment 2 Solutions

1 (a) Using $\mathcal{L}\{e^{at}f(t)\} = F(s-a)$,

$$\mathcal{L}\{e^{-3t}\,4t^3\} = F(s+3)$$

where $F(s) = \mathcal{L}\{4t^3\}$

$$= 4 \times \frac{3!}{s^4} = \frac{24}{s^4}$$

$$\therefore \mathcal{L}\{e^{-3t}\,4t^3\} = \frac{24}{(s+3)^4} \qquad (s > -3). \qquad 2$$

(b) $\mathcal{L}\{\cos^2(4t)\} = \mathcal{L}\{\tfrac{1}{2}(1+\cos 2t)\}$

$$= \tfrac{1}{2}\left(\mathcal{L}\{1\} + \mathcal{L}\{\cos 2t\}\right)$$

$$= \tfrac{1}{2}\left(\frac{1}{s} + \frac{s}{s^2+64}\right), \qquad (s>0) \qquad 3$$

(c) Using $\mathcal{L}^{-1}\{F(s-a)\} = e^{at}f(t)$,

$$\mathcal{L}^{-1}\left\{\frac{3}{(s+2)^3}\right\} = e^{-2t}f(t)$$

where $f(t) = \mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\left\{\frac{3}{s^3}\right\}$

$$= 3 \times \frac{t^2}{2}$$

$$\therefore \mathcal{L}^{-1}\left\{\frac{3}{(s+2)^3}\right\} = \frac{3}{2}e^{-2t}t^2. \qquad 2$$

(d) $\mathcal{L}^{-1}\left\{\dfrac{15s}{s^2+4s+29}\right\}$

$$= 15\,\mathcal{L}^{-1}\left\{\frac{(s+2)-2}{(s+2)^2+5^2}\right\}$$

$$= 15\,e^{-2t}\,\mathcal{L}^{-1}\left\{\frac{s-2}{s^2+5^2}\right\}, \quad \text{using } \mathcal{L}^{-1}\{F(s-a)\} = e^{at}f(t) = e^{at}\mathcal{L}^{-1}\{F(s)\}.$$

$$= 15e^{-2t}\left(\mathcal{L}^{-1}\left\{\frac{s}{s^2+5^2}\right\} - 2\,\mathcal{L}^{-1}\left\{\frac{1}{s^2+5^2}\right\}\right)$$

$$= 15e^{-2t}\left(\cos 5t - 2\times\tfrac{1}{5}\sin 5t\right)$$

$$= e^{-2t}(15\cos 5t - 6\sin 5t).$$ (3)

2. Let $Y(s) = \mathcal{L}\{y(t)\}$.

Taking the LT of each side of the DE:

$$s^2Y - sy(0) - y'(0) + 3[sY - y(0)] + 2Y = \frac{1}{s+2}$$

& using $y(0)=1$, $y'(0) = -3$

$$s^2Y - s + 3 + 3[sY - 1] + 2Y = \frac{1}{s+2}$$

$$(s^2+3s+2)Y - s = \frac{1}{s+2}$$

$$(s+1)(s+2)Y = \frac{1}{s+2} + s = \frac{1+s(s+2)}{s+2} = \frac{(s+1)^2}{s+2}$$ (2)

$$\therefore\ Y = \frac{(s+1)^2}{(s+1)(s+2)(s+2)} = \frac{s+1}{(s+2)^2}$$

$$\therefore Y = \frac{s+2-1}{(s+2)^2} = \frac{1}{s+2} - \frac{1}{(s+2)^2}$$

$$\therefore y(t) = \mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2}\right\}$$

$$= e^{-2t} - e^{-2t}\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} = e^{-2t} - e^{-2t}\,t$$

$$= e^{-2t}(1-t)$$ 3

3. Let $I(s) = \mathcal{L}\{i(t)\}$. Taking the LT of the DE:

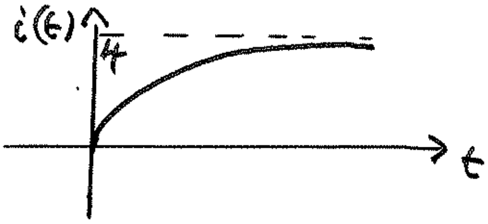
$$3[sI - i(0)] + 6I = \mathcal{L}\{e(t)\} = E(s).$$

Since $i(0)=0$, $$I = \frac{E(s)}{3(s+2)}.$$ 1

(a) $E(s) = \mathcal{L}\{24\} = \dfrac{24}{s}$.

$$\therefore I(s) = \frac{8}{s(s+2)} = 8\left[\frac{1/2}{s} + \frac{-1/2}{s+2}\right] = 4\left[\frac{1}{s} - \frac{1}{s+2}\right]$$

$$\therefore i(t) = 4(1-e^{-2t}).$$



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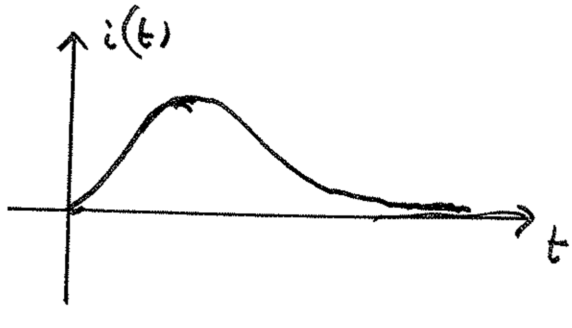
(b) $E(s) = \mathcal{L}\{9e^{-t}\} = \dfrac{9}{s+1}$

$$\therefore I(s) = \frac{3}{(s+1)(s+2)} = 3\left(\frac{1}{s+1} + \frac{-1}{s+2}\right)$$

$$\therefore i(t) = 3\left(\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\}\right)$$

$$= 3(e^{-t} - e^{-2t}).$$



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