

MATH2065: INTRO TO PDEs

Boundary Value Problems and Fourier Formulæ

$$\frac{d^2}{dx^2} \phi(x) = -\lambda \phi(x)$$

Boundary conditions	$\phi(0) = 0$ $\phi(L) = 0$	$\phi'(0) = 0$ $\phi'(L) = 0$	$\phi(-L) = \phi(L)$ $\phi'(-L) = \phi'(L)$
Eigenvalues λ_n	$\left(\frac{n\pi}{L}\right)^2$ $n = 1, 2, 3, \dots$	$\left(\frac{n\pi}{L}\right)^2$ $n = 0, 1, 2, 3, \dots$	$\left(\frac{n\pi}{L}\right)^2$ $n = 0, 1, 2, 3, \dots$
Eigenfunctions	$\sin \frac{n\pi x}{L}$	$\cos \frac{n\pi x}{L}$	$\sin \frac{n\pi x}{L}$ and $\cos \frac{n\pi x}{L}$
Series	$f(x) = \sum_{n=1}^{\infty} B_n \sin \frac{n\pi x}{L}$	$f(x) = \sum_{n=0}^{\infty} A_n \cos \frac{n\pi x}{L}$	$f(x) = \sum_{n=0}^{\infty} a_n \cos \frac{n\pi x}{L}$ $+ \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}$
Coefficients	$B_n = \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi x}{L} dx$	$A_0 = \frac{1}{L} \int_0^L f(x) dx$ $A_n = \frac{2}{L} \int_0^L f(x) \cos \frac{n\pi x}{L} dx$	$a_0 = \frac{1}{2L} \int_{-L}^L f(x) dx$ $a_n = \frac{1}{L} \int_{-L}^L f(x) \cos \frac{n\pi x}{L} dx$ $b_n = \frac{1}{L} \int_{-L}^L f(x) \sin \frac{n\pi x}{L} dx$
Coefficients obtained by	Half-Range Fourier Sine Series	Half-Range Fourier Cosine Series	Standard Fourier Series