MATH2065: INTRO TO PDEs

Summer School 2012

Tutorial Questions 1

1. Show that the ordinary differential equation
$$\frac{dx}{dt} = \mu x, \quad \mu = \text{constant}$$
has the general solution
$$x = A\, e^{\mu t},$$
where A is a constant.

   Hence solve

   (a) $dx/dt = 3x, \quad x = 2$ when $t = 0$.
   (b) $dy/dz = -5y, \quad y = 10$ when $z = 1$.
   (c) $dx/dt = 4 - 3x, \quad x = 0$ when $t = 0$.[Hint: let $X = 4 - 3x$]

2. Find the general solution to each of the following ordinary differential equations ($y' = dy/dx$, etc.)

   (a) $y' - 7\,y = 0$
   (b) $y'' + y' - 2\,y = 0$
   (c) $y'' - 3\,y' = 0$
   (d) $y'' - 4\,y = 0$
   (e) $y'' + 4\,y = 0$
   (f) $y'' + 6\,y' + 9\,y = 0$
   (g) $y'' - 6\,y' + 25\,y = 0$
   (h) † $y''' + 2\,y'' - y' - 2\,y = 0$

3. Determine the solution to each of the following initial value problems.

   (a) $y' - 7\,y = 0 \;\;;\;\; y(0) = 1$
   (b) $y'' - 3\,y' = 0 \;\;;\;\; y(0) = 0\;,\; y'(0) = 1$
   (c) $y'' + 6\,y' + 9\,y = 0 \;\;;\;\; y(0) = 0\;,\; y'(0) = 1$
   (d) $y'' - 6\,y' + 25\,y = 0 \;\;;\;\; y(0) = 0\;,\; y'(0) = 4$

4. Verify that $y_1(t) = t^3$ and $y_2(t) = t^3 \ln t$ both satisfy the differential equation
$$t^2\, y'' - 5\, t\, y' + 9\, y = 0$$
in the domain $\{t > 0\}$. Hence, make a guess at the *general solution* to this second-order linear ordinary differential equation.

5. (a) Write down the definitions for the functions $\cosh x$ and $\sinh x$ in terms of exponential functions. Sketch these functions.
   (b) Consider the expression $y(x) = C_1 e^{px} + C_2 e^{-px}$ for some constant p, and suppose C_1 and C_2 are arbitrary constants. Show that this can be written alternatively as
$$y(x) = D_1 \cosh px + D_2 \sinh px$$
where D_1 and D_2 are arbitrary constants.