MATH2065: INTRO TO PDEs

Summer School 2012

Tutorial Questions 2

Questions marked with the dagger symbol † are intentionally more challenging.

1. Use the method of undetermined coefficients to find a particular solution to each of the following inhomogeneous differential equations:

   (a) $y'' - 4\,y = e^x$

   (b) $y'' - 9\,y = x + 18$

   (c) $y'' - y' + 2\,y = -2\sin x$

   (d) $y'' + y = x^2$

   (e) $y'' - 9\,y = 2e^{3x}$

2. Solve the following initial value problems.

   (a) $y'' + y' - 6y = 5e^x$, with initial conditions $y(0) = 0$ and $y'(0) = 2$

   (b) $y'' + 3y' + 2y = xe^{4x}$, with initial conditions $y(0) = 0$ and $y'(0) = 1$

   (c) $y'' - 4y' + 3y = 2\sin x$, with initial conditions $y(0) = 1$ and $y'(0) = 0$

3. Show that if $y_1(x)$ is a solution of $y'' + a\,y' + b\,y = f_1(x)$ and if $y_2(x)$ is a solution of $y'' + a\,y' + b\,y = f_2(x)$, then the function $y_1(x) + y_2(x)$ is a solution of

   $$y'' + a\,y' + b\,y = f_1(x) + f_2(x)\,.$$

   Use this result to obtain a particular solution of the following ODEs.

   (a) $3\,y'' + 10\,y' + 3\,y = x + \cos x$

   (b) $y'' + 5\,y' + 6\,y = \sin x + 2\,e^x$

   (c) $y'' - 4\,y' + 3\,y = e^x + e^{2x}$

4. **(Resonance)** Consider a block and spring with mass m, spring constant k, damping coefficient γ, and imposed forcing $F(t)$. Recall that its motion can be represented by

   $$m\,\ddot{x} + \gamma\,\dot{x} + k\,x = F(t)\,.$$

   where $x(t)$ represents the position of the block measured from its equilibrium. For the purposes of this problem, consider the particular case where $m = 1$, $\gamma = 0$, $k = 4$ and $F(t) = \cos\omega t$, where the frequency ω is a positive constant.

   (a) Recall that the solution $x(t)$ can be represented as $x(t) = x_h(t) + x_p(t)$, where x_h and x_p are respectively the homogeneous and the particular solutions to the ODE. Determine $x_h(t)$.

   (b) Determine $x_p(t)$. [Hint: does your solution work for all positive ωs, or do you have to consider some special cases?]

   (c) † Based on your solutions to (a) and (b) above, describe the behaviour of the block at large times. Is there a "special" value of the forcing frequency ω at which something different happens?