

# MATH2065: INTRO TO PDES

*Summer School 2013*

## Tutorial Questions 3

Questions marked with the dagger symbol † are intentionally more challenging.

1. Find the Laplace transforms of each of the following functions.

(a)  $t^3 e^{-2t}$

(b)  $t \sin 4t$

(c)  $H(t - 3)$

(d)  $e^{3t} \sin 4t$

(e)  $t e^{-4t} \cos 6t$

(f)  $f(t) = \begin{cases} 0 & \text{if } t < 5 \\ t^2 & \text{if } 5 < t < 8 \\ 0 & \text{if } 8 < t \end{cases}$

(g)  $(t - 1)^4 H(t - 1)$

2. Using a Laplace transform table (provided in lectures), determine the inverse Laplace transforms of each of the following functions.

(a)  $\frac{1}{s^2 + 4}$

(b)  $\frac{e^{-3s}}{s^2 + 4}$

(c)  $(s - 4)^{-7}$

(d)  $\frac{s}{s^2 + 8s + 7}$

(e)  $\frac{s}{(s^2 + 1)(s^2 + 4)}$

(f) †  $\frac{(s + 2)(1 - 5e^{-4s})}{s^3 + 9s}$

3. Prove the following properties concerning Laplace transforms. Here,  $F(s)$  is the Laplace transform of a function  $f(t)$ .

(a)  $\mathcal{L}\{(-t)^n f(t)\} = \frac{d^n}{ds^n} F(s)$  for nonnegative integers  $n$

(Note: we have proven in class that  $\mathcal{L}\{-tf(t)\} = F'(s)$  – does this help?)

(b)  $\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$  for nonnegative integers  $n$

(c)  $\mathcal{L}\{f''(t)\} = s^2 F(s) - s f(0) - f'(0)$

(Note: a result we have shown in class which may be relevant is that  $\mathcal{L}\{f'(t)\} = s F(s) - f(0)$ .)

4.  $\frac{dy}{dt} + y = 1$  with  $y(0) = 2$ .

5.  $\frac{d^2y}{dt^2} + 4y = \sin t$  with  $y(0) = 0$  and  $\frac{dy}{dt}(0) = 0$ .