

# MATH2065: INTRO TO PDES

Summer School 2012

## Tutorial Questions 7

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1. Solve Laplace's equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

for  $\phi(x, y)$  in the square  $0 < x < L$ ,  $0 < y < L$ , subject to the boundary conditions

$$\begin{aligned}\phi(x, 0) &= 0 && \text{for } 0 \leq x \leq L, \\ \phi(x, L) &= 0 && \text{for } 0 \leq x \leq L, \\ \phi(0, y) &= 0 && \text{for } 0 \leq y \leq L, \\ \phi(L, y) &= f(y) && \text{for } 0 \leq y \leq L,\end{aligned}$$

where  $f$  is a given function. [This models the electrostatic potential in a square plate, in which three sides of the plate are earthed, and the fourth side is subjected to an imposed potential variation.]

2. Solve the Laplace equation inside the rectangle  $0 \leq x \leq L$ ,  $0 \leq y \leq H$  with boundary conditions  $\frac{\partial u}{\partial x}(0, y) = 0$ ,  $\frac{\partial u}{\partial x}(L, y) = 0$ ,  $u(x, 0) = 0$ ,  $u(x, H) = f(x)$ .

3. † Solve the Laplace equation *outside* the circular disk ( $r \geq a$ ) subject to the boundary conditions

(a)  $u(a, \theta) = \ln 2 + 4 \cos 3\theta$

(b)  $u(a, \theta) = f(\theta)$