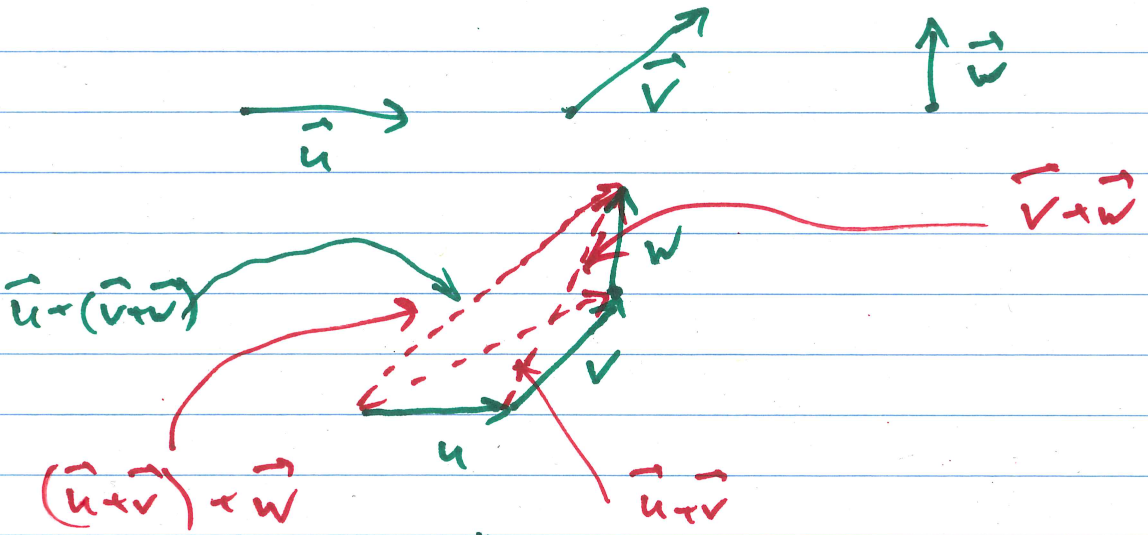
L.2

**Claim:** $$(\vec{u}+\vec{v})+\vec{w} = \vec{u}+(\vec{v}+\vec{w})$$

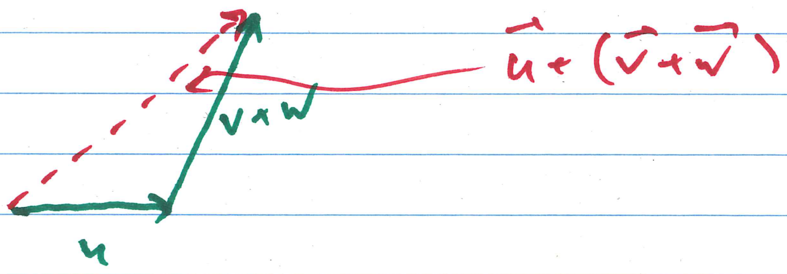
for all vectors $\vec{u},\vec{v},\vec{w}$.

(associative rule)

**Geometric Proof:**



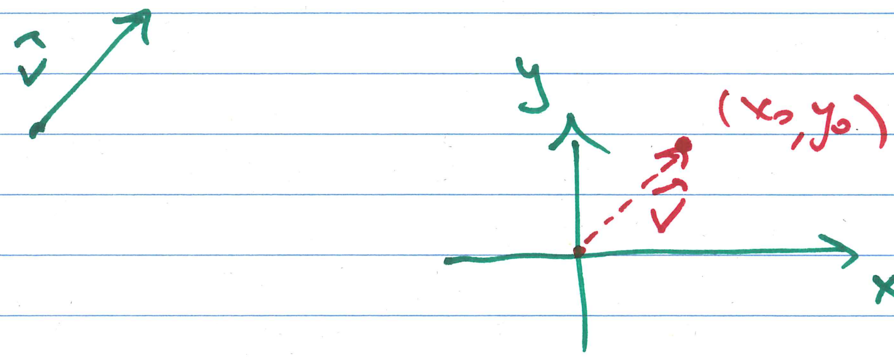

$\Downarrow$




How do we add vectors algebraically?

In the plane:

Every vector $\longleftrightarrow$ $(x_0, y_0)$



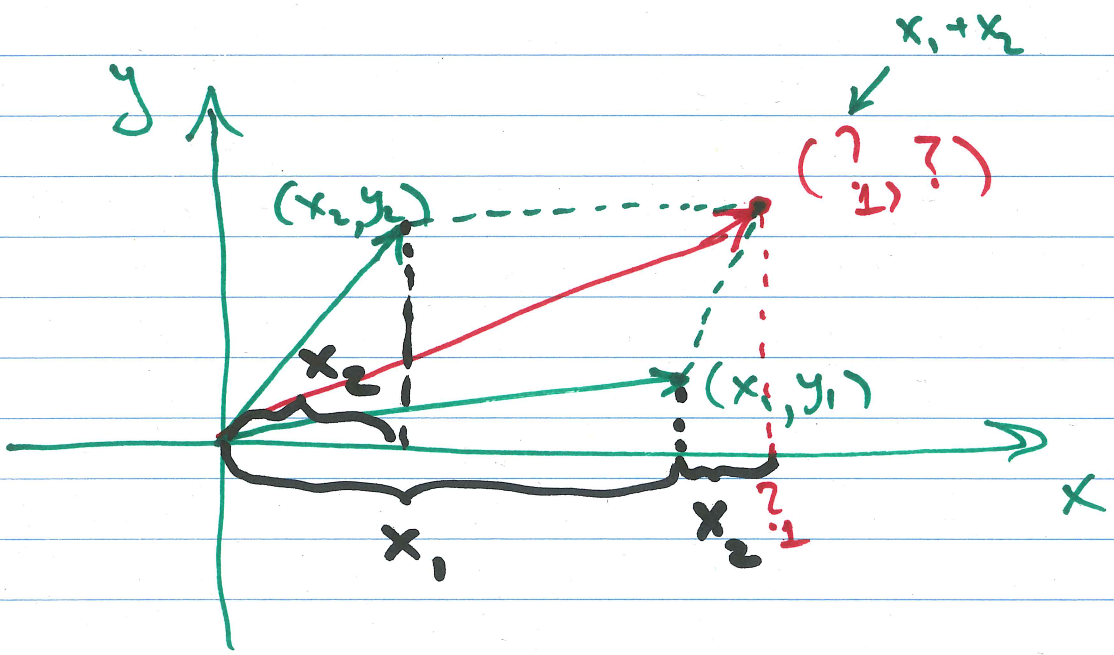

<u>Claim:</u>

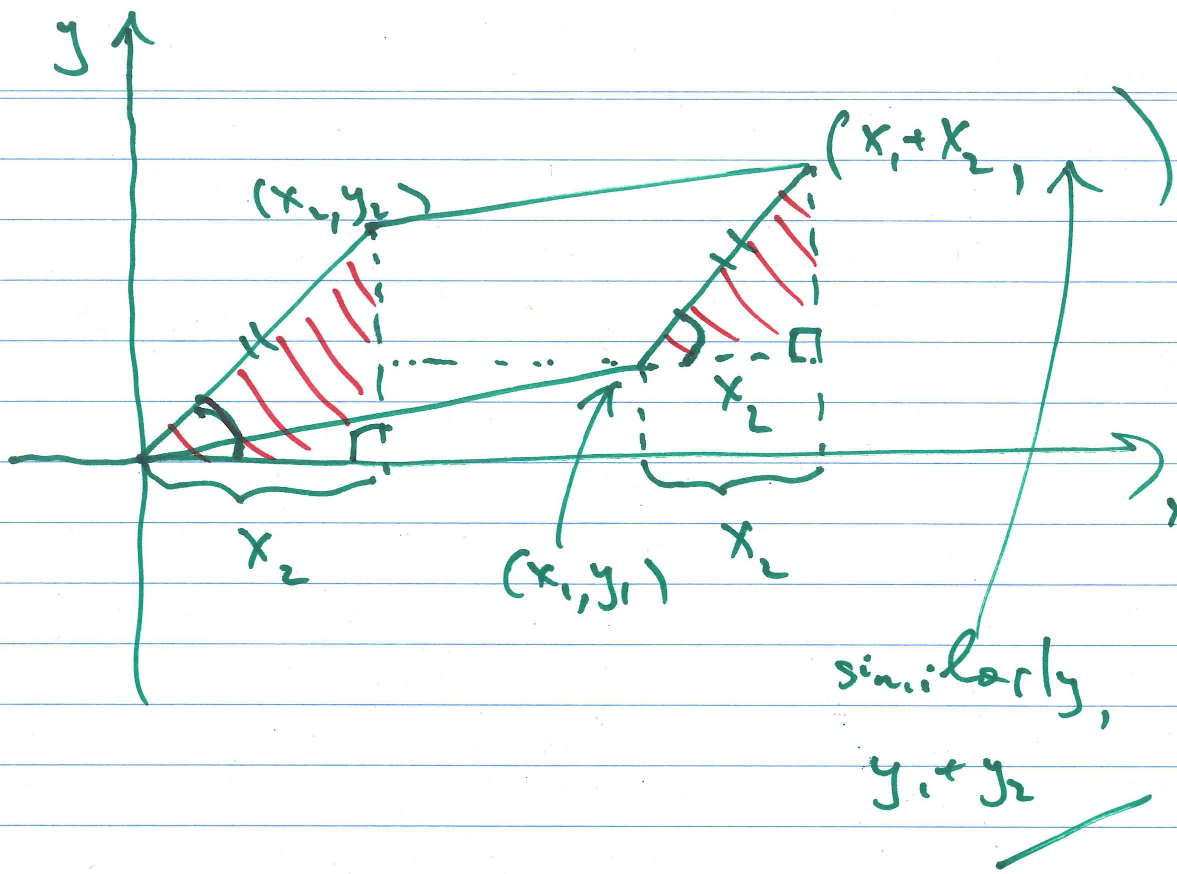
$$(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

$$\alpha \cdot (x_0, y_0) = (\alpha \cdot x_0, \alpha \cdot y_0)$$

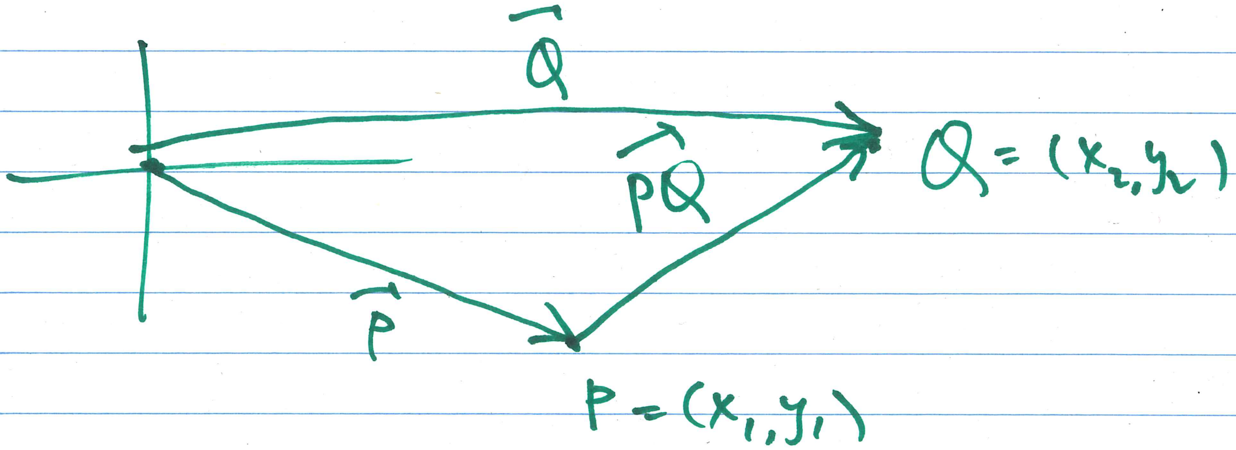
for every $\alpha$ - real #.

"Proof":



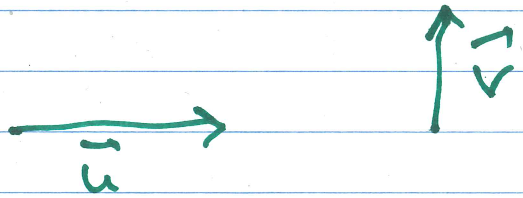



<u>We can explain:</u>

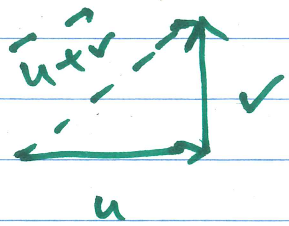



then $\vec{PQ} = (x_2 - x_1, y_2 - y_1) \left(= \vec{Q} - \vec{P}\right)$

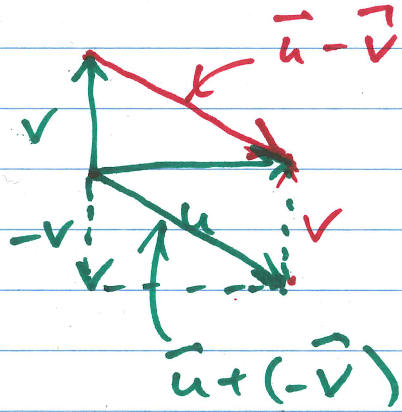
What is the difference of two vectors?



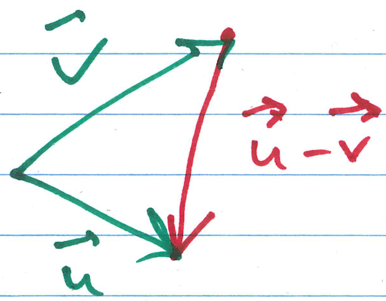

$\vec{u}+\vec{v}$ :




$$\vec{u}-\vec{v} \doteqdot \vec{u}+(-1)\cdot\vec{v}$$

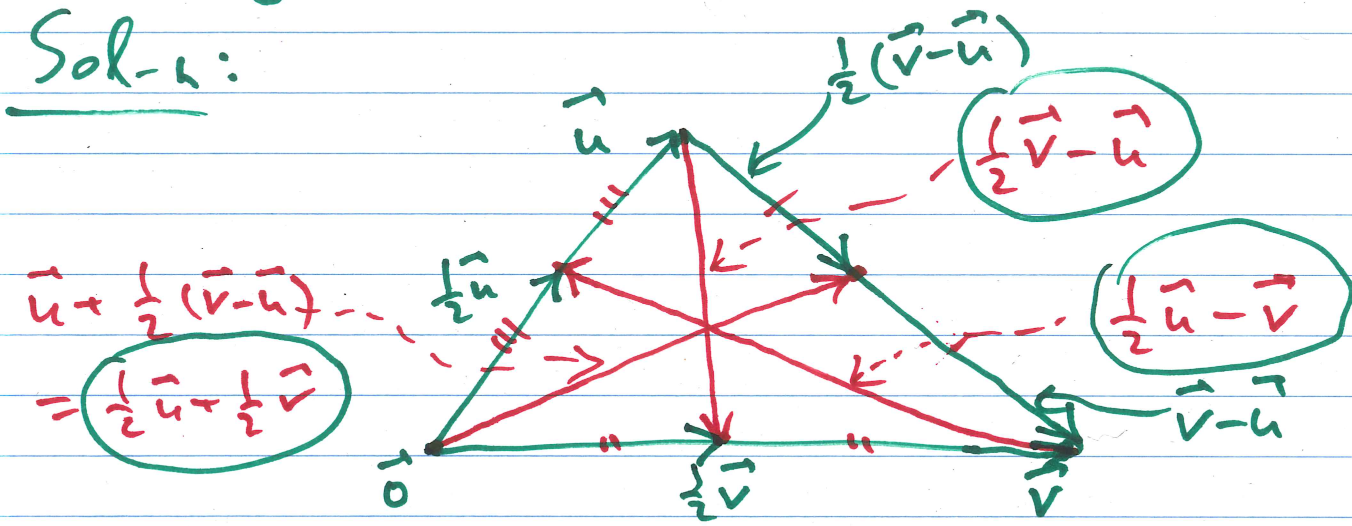



We showed:

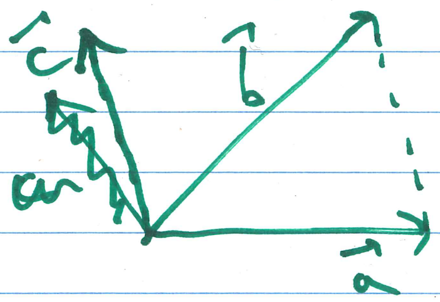


**Exercise:** Show that it is possible to move in a parallel way the medians of any triangle to create a triangle.

**Sol-n:**




What does it mean that three vectors $\vec{a}, \vec{b}, \vec{c}$ can form a triangle?




$\leftarrow \quad \vec{a}+\vec{c}=\vec{b}$

$\Leftrightarrow \quad \vec{a}+\vec{c}-\vec{b}=\vec{0}$

$$\Leftrightarrow \quad \pm\vec{a} \pm \vec{b} \pm \vec{c} = \vec{0}.$$

And we definitely have:

$$\left(\tfrac{1}{2}\vec{u}-\vec{v}\right)+\left(\tfrac{1}{2}\vec{u}+\tfrac{1}{2}\vec{v}\right)+\left(\tfrac{1}{2}\vec{v}-\vec{u}\right)=\vec{0}$$