L.4

The second eq-n is:

$$\tfrac12 v + \gamma\left(u - \tfrac12 v\right) = \tfrac{\beta}{2}(u+v)$$

But $\gamma = \frac13$ (from previous part)

$$\Big(\underset{\gamma}{\underset{\parallel}{\tfrac13}} - \tfrac{\beta}{2}\Big)u + \Big(\tfrac12 - \underset{\frac{\gamma}{2}}{\underset{\parallel}{\tfrac16}} - \tfrac{\beta}{2}\Big)v = \vec{0}$$

u & v are non-parallel $\Rightarrow$

$$\begin{cases} \frac13 - \frac{\beta}{2} = 0 & \Rightarrow \quad \frac{\beta}{2} = \frac13 \Rightarrow \beta = \frac23 \\ \frac12 - \frac16 - \frac{\beta}{2} = 0 & \Rightarrow \quad \text{the same } \beta = \frac23 . \end{cases}$$

Halleluya – for $\alpha = \frac13$, $\beta = \frac23$, $\gamma = \frac13$ the system of eq-ns is satisfied.

$\Rightarrow$ Indeed, all the medians of a triangle intersect in one pt.

<u>Cor</u>: The intersection pt of the medians divides every median in the ratio $1:2$ (the part adjacent to the edge is shorter than the other part)

Def: (linear independence of vectors)
We say that vectors $v_1, \ldots, v_n$ are linearly independent if for any linear combination

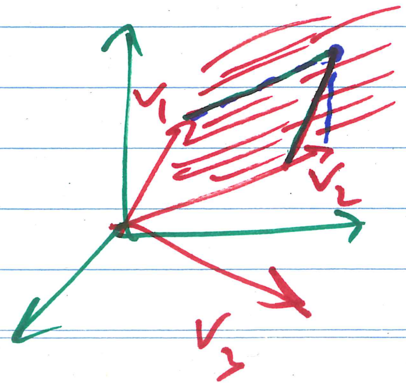
$$\alpha_1 v_1 + \alpha_2 \cdot v_2 + \ldots + \alpha_n v_n = \vec{0}$$

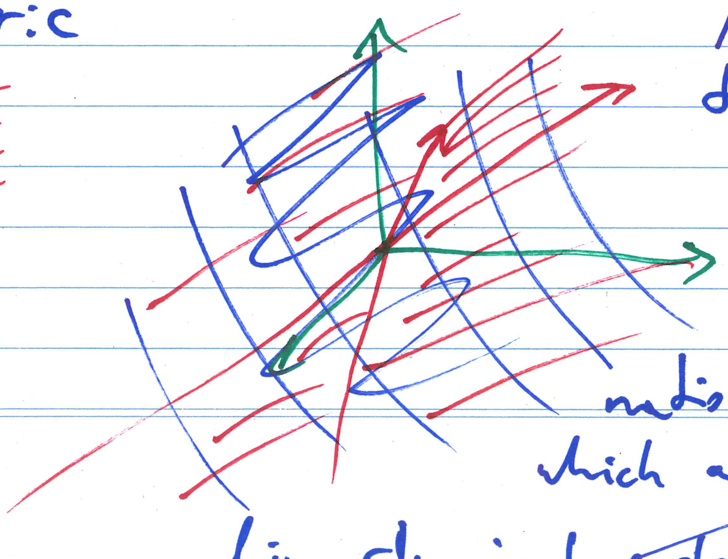
$$\Rightarrow \quad \alpha_1 = \alpha_2 = \ldots = \alpha_n = 0$$

Claim: Two vectors are linearly independent $\Leftrightarrow$ they are not parallel.

Exercise: Show that any 4 vectors in 3-dimensional space are NOT linearly independent (linearly dependent)

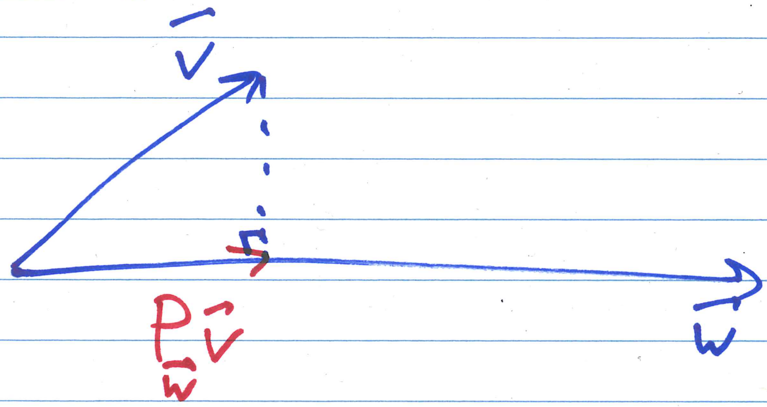
Pf: Geometric





"No space" for 4-th vector s.t. it is not a linear combination of v_1, v_2, v_3 which are linearly independent.

Projection :



We can geometrically project vector $\vec{v}$ on $\vec{w}$. (if $\vec{w} \neq \vec{0}$)

Goal: To find $P_{\vec{w}}\vec{v}$ by looking at its geometric properties.

Properties of $P_w v$:

- $P_w v$ is on the line generated by w. In other words, there exists λ real s.t.

$$P_w v = \lambda \cdot w.$$

- The angle between $P_w v$ & $v - P_w v$ is $90°$. In other words,

$P_w v$ is orthogonal to $v - P_w v$.

We will find λ (in $P_w v = \lambda v$) by use of Pythagoras thm:

$$\underbrace{|v|^2}_{} = \underset{P_w v}{|\lambda w|^2} + \underset{P_w^{\perp} v}{|v - \lambda w|^2}$$

Let's $v = (a, b)$, $w = (c, d)$ $\Rightarrow$ $v - \lambda w = (a - \lambda c, b - \lambda d)$

Then $|v|^2 = a^2 + b^2$

$$|\lambda w|^2 = \lambda^2 (c^2 + d^2)$$

$$|v - \lambda w|^2 = (a - \lambda c)^2 + (b - \lambda d)^2$$

And we have:

$$\underset{|v|^2}{(a^2 + b^2)} = \lambda^2 (c^2 + d^2) + (a - \lambda c)^2 + (b - \lambda d)^2$$

$$0 = \cancel{2} \lambda^2 (c^2 + d^2) - \cancel{2} \lambda (ac + bd)$$

$$\lambda \cdot (\lambda (c^2 + d^2) - (ac + bd)) = 0$$

$\lambda = 0$ (not interesting) $\qquad \lambda = \dfrac{ac + bd}{c^2 + d^2}$

$$\Rightarrow \quad \lambda = \frac{ac+bd}{c^2+d^2} = \frac{ac+bd}{|w|^2} = \frac{\vec{v}\cdot\vec{w}}{|w|^2}$$

Conclusion:

$$P_w v = \frac{\vec{v}\cdot\vec{w}}{|\vec{w}|^2}\cdot\vec{w}$$