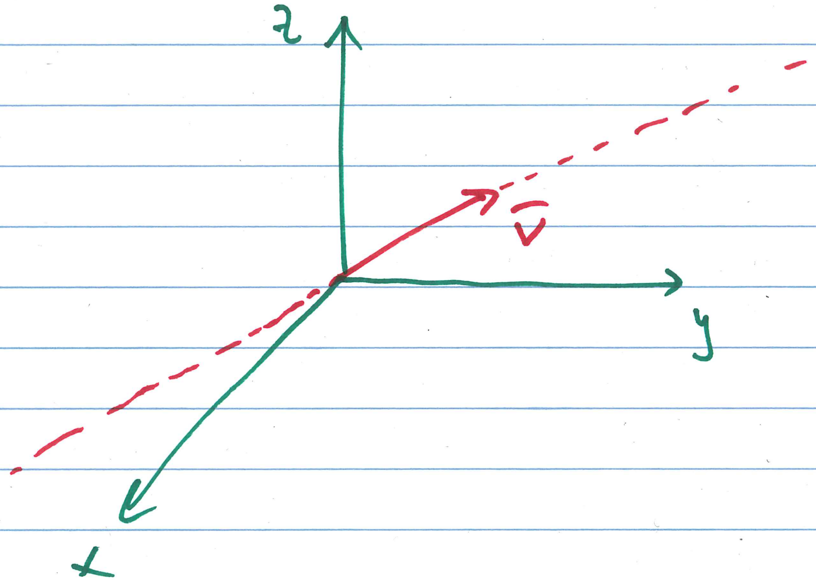
L.9

Lines in Space

Q: What is the line in space (plane)?

A: Case 1: Line through the origin in the direction of $\vec{v} = (a, b, c)$.



The line which form all vectors parallel to $\vec{v}$ $\Rightarrow$ all vectors of the form

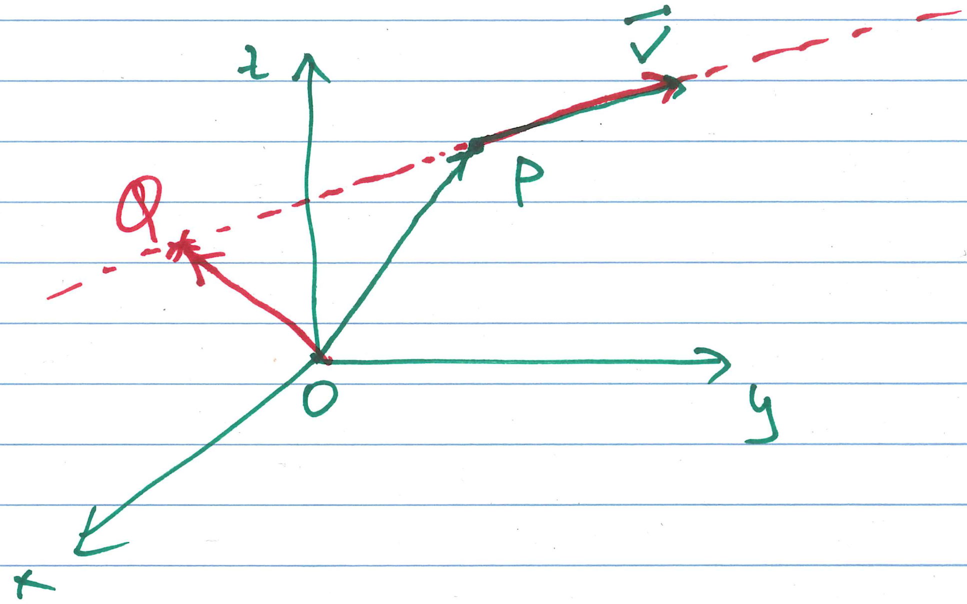
$\lambda \vec{v}$, λ - real number

Usually, we replace λ by t (denotes time):

$$\boxed{(x, y, z) = t \cdot \vec{v} = (ta, tb, tc)}$$

In the plane: $\boxed{(x, y) = (ta, tb)}$

Case 2 (general): Line passing through pt $P=(x_0,y_0,z_0)$ in the direction of $\vec{v}=(a,b,c)$



All the points in the space which can be written in the form:

$$(x,y,z)=\vec{OP}+t\cdot\vec{v} = (x_0,y_0,z_0)+t(a,b,c)$$

$$=\boxed{x_0\vec{i}+y_0\vec{j}+z_0\vec{k}+t(a\vec{i}+b\vec{j}+c\vec{k})}$$

Parametric vector eq-n of a line

Or, we can write the latter in the form:

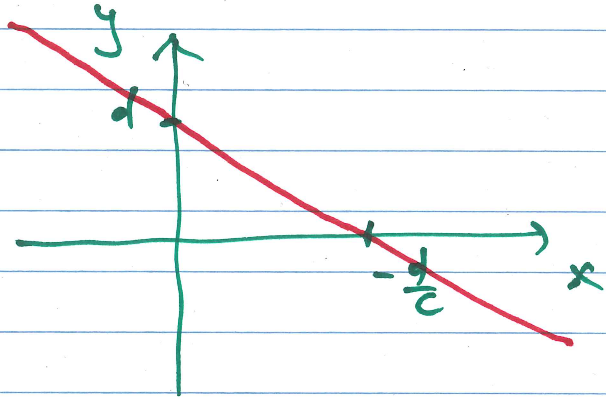
$$\boxed{\begin{cases} x=x_0+ta \\ y=y_0+tb \\ z=z_0+tc \end{cases}}$$

Parametric scalar eq-n of a line

Q: Is the last eq-n (in the plane) ~~the~~ the same as the eq-n

$$(*) \quad y = cx + d, \text{ for some } c, d \text{ real numbers}$$

A:



Recall, the parametric scalar eq-n in the plane is of the form:

$$(**) \quad \begin{cases} x = x_0 + ta \\ y = y_0 + tb \end{cases}$$

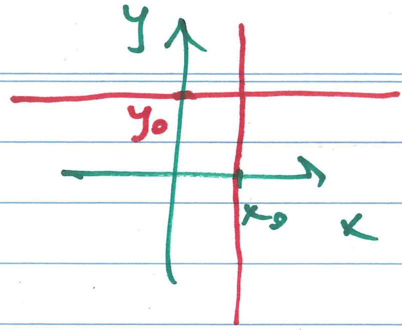
(x_0, y_0), (a, b) are fixed, t changes.

We will connect eq-ns $(*)$ & $(**)$:

If $a \neq 0$, $b \neq 0$: $\quad t = \dfrac{x - x_0}{a} = \dfrac{y - y_0}{b}$

$$\Rightarrow \frac{x - x_0}{a} = \frac{y - y_0}{b} \Rightarrow y = \underbrace{\frac{b}{a}}_{c} x + \underbrace{\left(y_0 + \left(-\frac{x_0 b}{a}\right)\right)}_{d}$$

If $a=0$: $x = x_0$



If $b=0$: $y = y_0$

How do we get eq-n (**) if we are given a line by eq-n (*) ?

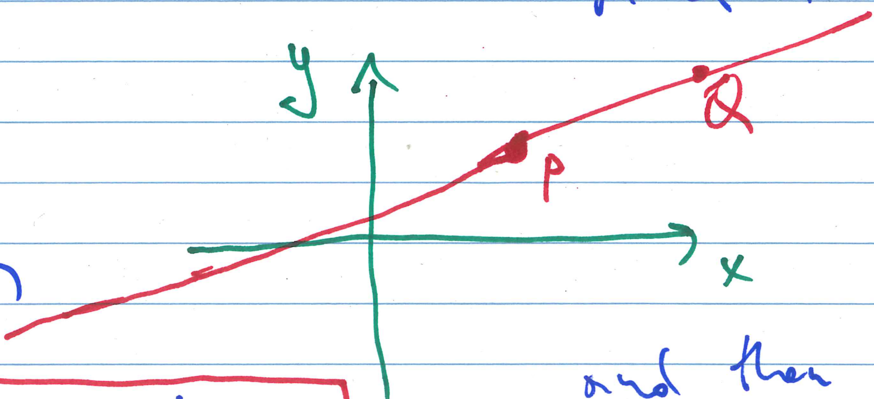
Ex: Find eq-n of a line in parametric form if it is given by the eq-n

$$y = 3x - 2$$

A.: We will find a pt lying on the line & vector of the direction:

$P = (0,-2)$ is on the line

Vector of direction $= ?$ just find another pt Q on the line



$Q = (1,1)$

$\Rightarrow \vec{PQ} = (1,3)$

$\Rightarrow \boxed{(x,y) = (0,-2) + t(1,3)}$

and then $\vec{PQ}$ is vector of direction.

Attention: The eq-n of a line is not unique!!!

Ex: For the line $y = 3x - 2$, we already found that one of its parametric eq-ns is

$$(x,y) = (0,-2) + t(1,3).$$

But, we could choose $P' = (2,4)$,

$Q' = (4, 10) \Rightarrow \overrightarrow{P'Q'} = \cancel{(1,3)}\ (2,6)$

$\Rightarrow$ eq-n of the line is

$$\boxed{(x,y) = (2,4) + t(2,6)}$$

Q: Do we have an analog of eq-n (*) in the space?

$$\boxed{y = cx + d}$$

A.: NO, if we could then

$z = ax + by + c$ is eq-n of a plane in the space we have two free variables!!!

Analog of ⊛ in the space:

Cartesian eq-ns of a line

$$\begin{cases} x = x_0 + ta \\ y = y_0 + tb \\ z = z_0 + tc \end{cases}$$

If $a \neq 0,\ b \neq 0,\ c \neq 0$:

$$t = \boxed{\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}}$$

Cartesian eq-ns of a line